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Volume III

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ADVANCED COMPOSITES DATA FOR AIRCRAFT STRUCTURAL DESIGN

Volume III: Theoretical Methods

L. M. Lackman
T. H. Lin
D. Y. Konishi
J. W. Davidson

LOS ANGELES DIVISION
NORTH AMERICAN ROCKWELL

TECHNICAL REPORT AFML-TR-70-58, VOLUME III

DECEMBER 1970

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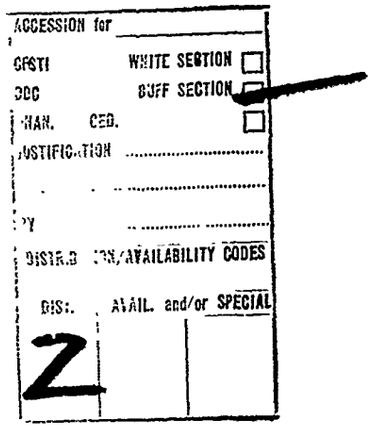
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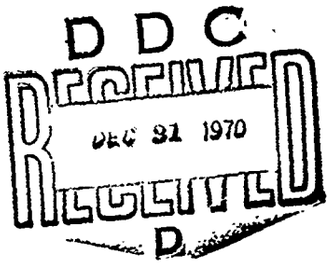


AFML-TR-70-58
Volume III

ADVANCED COMPOSITES DATA FOR AIRCRAFT STRUCTURAL DESIGN

Volume III: Theoretical Methods

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FOREWORD

This report was prepared by the Los Angeles Division of North American Rockwell Corporation under Contract F33615-68-C-1489 for the Advanced Composites Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. Mr. R. L. Rapson (AFML/LC) was the Air Force Project Engineer and Dr. L. M. Lackman was the North American Rockwell Program Manager.

The authors of Volume III are Dr. L. M. Lackman, Mr. D. Y. Konishi, Dr. T. H. Lin, and Mr. J. W. Davidson, who must be credited collectively for this volume, since, for the most part, the author of each section was supported substantially by some or all of the others. In addition, Dr. A. Caputo contributed to the development of the thermoelastic relationships reported in section VII. Mr. G. H. Arvin, as Editor in Chief, was responsible for the organization and continuity of this volume.

This report was submitted by the authors on 14 May 1970 for publication as a technical report.

This report has been reviewed and is approved.

Robert C. Tomashot
Technical Area Manager
Advanced Composites Division

ABSTRACT

This volume summarizes that portion of the program concerned with the development of theoretical methods of analysis for orthotropic laminated composite structure. An appendix to this volume presents a number of computer programs that were generated in conjunction with these theoretical developments.

The main body of this volume is comprised of independent sections devoted to the following topics:

Extensional and flexural elastic constants

Flat plates

Orthotropic sandwich plates and shells

Stiffened skin construction

Minimum weight analysis

Thermoelastic relationships

Joint analysis

The appendix to this volume presents, for each of the included computer programs, an abstract, data input instructions, output description, and a typical example problem. The listed programs cover the following subjects:

Calculation of laminate extensional and flexural elastic constants.

Honeycomb sandwich panels under in-plane biaxial loading, in-plane shear loading, normal pressure, or certain combinations of these.

Hat- and Z-stiffened panel stability and stiffness.

Laminate thermoelastic analysis.

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TABLE OF CONTENTS

Section		Page
I	INTRODUCTION	1
II	EXTENSIONAL AND FLEXURAL ELASTIC CONSTANTS	5
	Stress-Strain Relations of a Lamina in Generalized Plane Stress	5
	Sectional Forces and Moments of a Laminate in Terms of Displacements	7
	Laminate Extensional Constants	9
	D_{ij}'s of Some Commonly Used Laminates	11
	Calculations of Extensional and Flexural Constants	15
III	FLAT PLATES	29
	Equations of Motion	29
	Pressure Panels	37
	Uniaxial and Biaxial Compression	38
	Natural Frequencies	42
IV	ANALYSIS OF ORTHOTROPIC SANDWICH PLATES AND SHELL STRUCTURES	43
	Constitutive Relations	43
	Compression Stability	50
	Uniformly Distributed Pressure and Edge Compression	61
	Shear Stability of Flat Sandwich Panels	69
V	STIFFENED SKIN CONSTRUCTION	75
	Constitutive Relationships	75
	Zee Stiffeners	82
	Hat Stiffeners	84
VI	MINIMUM WEIGHT ANALYSIS	87
	General	87
	Zee Stiffeners	88
	Hat Stiffeners	90
VII	THERMOELASTIC RELATIONSHIPS	91

Section		Page
VIII	JOINT ANALYSIS	99
	Single-Lap Adhesive Joints	99
	Continuous Taper Scarf Joint	100
	Double-Lap Adhesive Joint	101
	Double-Lap Variable Stiffness Adhesive Joint	106
REFERENCES		115
APPENDIX	COMPUTER PROGRAMS	117

LIST OF ILLUSTRATIONS

Figure	Title	Page
1	Typical Lamina	5
2	Lamina and Laminate Axes	6
3	Laminate of N Laminae	8
4	Positive Direction of Internal Forces and Moments	8
5	Typical Symmetrical Laminate	11
6	Laminate E_x vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT	17
7	Laminate E_x vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at 350° F	18
8	Laminate G_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT and 350° F	19
9	Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT	20
10	Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at 350° F	21
11	Laminate E_x vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT	22
12	Laminate E_x vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F	23
13	Laminate E_y vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT	24
14	Laminate E_y vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F	25
15	Laminate G_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT and 350° F	26
16	Laminate ν_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT	27
17	Laminate ν_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F	28
18	Plate Element Subjected to Force and Moment Components	30
19	Thin and Thick Laminate Deformation	31
20	D_{16}/D_{11} vs Number of Plies for Several Laminate Orientations	32
21	D_{26}/D_{11} vs Number of Plies for Several Laminate Orientations	33
22	D_{26}/D_{22} vs Number of Plies for Several Laminate Orientations	34
23	D_{16}/D_{22} vs Number of Plies for Several Laminate Orientations	35
24	Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (RT)	40
25	Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (350° F)	41
26	Section of a Sandwich Plate	44
27	Positive Direction of Sectional Forces and Moments	44

Figure	Title	Page
28	Rotations Due to Shear.	44
29	Buckling Coefficient K_x vs V for Simply Supported Flat Sandwich Panels (RT).	52
30	Buckling Coefficient K_x vs V for Simply Supported Flat Sandwich Panels (350° F).	53
31	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 1 (RT).	54
32	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 2 (RT).	55
33	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 3 (RT).	56
34	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 1 (350° F).	57
35	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 2 (350° F).	58
36	Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 3 (350° F).	59
37	Sandwich Panel Subjected to Normal Load and Edge Compression.	62
38	Parameter $w\,\xi/qb^4$ vs V for Simply Supported Honeycomb Sandwich Panels	68
39	Typical Honeycomb Sandwich Panel Subjected to In-plane Shear Loading	70
40	Typical Stiffened Sheet	75
41	Typical Zee Stiffener	82
42	Typical Hat Stiffener	90
43	Coordinates of i^{th} Ply in Natural Lamina and Laminate Systems	92
44	Relationship Between Laminae and Laminate Coordinate Systems.	92
45	Typical Single Lap Joint.	99
46	Typical Scarf Joint	100
47	Typical Double Lap Joint.	101
48	Typical Load Transfer Through Adhesive.	102
49	Typical Variable Stiffness Adhesive Double-Lap Joint.	106
50	Double-Lap Variable Adhesive - Typical Distribution of $\bar{f}_a^s$ Versus x/L_a	113

LIST OF TABLES

Table	Title	Page
I	Internal Moment and Shear Coefficients for Simply Supported Panels With No In-Plane Loads.	67
II	Maximum Normal Deflection and Internal Moments and Shears for Simply Supported Panels With $N_x/(N_x)_{cr} = 0.333$	69

LIST OF SYMBOLS

a	- length dimension (in.), especially rectangular panel
A_{ij} $(i,j=1,2,6)$	- extensional rigidities (lb/in.)
b	- width dimension (in.), especially width of compression panel normal to load
b_s	- stiffener spacing (in.)
c	- honeycomb sandwich core depth (in.)
D_{ij} $(i,j=1,2,6)$	- flexural rigidities (lb/in.)
$DQ_x = cG'_{cx}$	- honeycomb sandwich shear stiffness along X axis
$DQ_y = cG'_{cy}$	- honeycomb sandwich shear stiffness along Y axis
d_s	- width of stiffener element (in.)
E_L	- Young's modulus of laminae parallel to filament direction ($lb/in.^2$)
E_T	- Young's modulus of laminae transverse to filament direction ($lb/in.^2$)
E_x	- Young's modulus of laminate along X reference axis ($lb/in.^2$)
E_y	- Young's modulus of laminate along Y reference axis ($lb/in.^2$)
f_s	- width of stiffener element (in.)
G_{LT}	- shear modulus of laminae in TL plane ($lb/in.^2$)
G_{xy}	- shear modulus of laminate in XY plane ($lb/in.^2$)

G'_{cx} - shear modulus of honeycomb sandwich core in XZ plane ($lb/in.^2$)

G'_{cy} - shear modulus of honeycomb sandwich core in YZ plane ($lb/in.^2$)

h_i - thickness of i^{th} lamina (in.)

h_s - stiffener depth (in.)

I_1 - invariant defined by equation 83

I_2 - invariant defined by equation 84

k - (1) buckling coefficient
(2) coefficient of thermal conductivity ($BTU/ft^2/hr/in./°F$)

k_x, k_y - plate compression buckling coefficients

k_s - plate shear buckling coefficient

L - cylinder, beam, or column length (in.)

M - mass per unit area of laminate

m - number of half-wave lengths

M_x, M_y, M_{xy} - bending and twisting moment components (in.-lb/in.)

N - number of laminae in a laminate

n - (1) number of half-wave lengths
(2) number of laminae in a laminate
(3) shear stiffness ratio $\frac{DQ_y}{DQ_x}$

N_x, N_y, N_{xy} - force components (lb/in.)

P - applied load (lb)

p_1 - number of plies in a set which are at 0°

p_2 - number of plies in a set which are at 90°

p_θ - half the number of plies in a set which are at $\pm\theta°$

Q_x, Q_y - ' ar force parallel to Z axis of sections of a plate :pendicular to the X and Y axes, respectively (lb/in.)

Q_{ij}, T_{ij} (i,j=1,2,6) - lamina elastic constants in natural coordinate system

$\bar{Q}_{ij}$ (i,j=1,2,6) - lamina elastic constants in laminate coordinate system

R - radius

r - honeycomb core shear modulus ratio $\frac{G'_{cx}}{G'_{cy}}$

T - temperature (°F)

t - thickness (in.)

t_f - honeycomb sandwich face sheet thickness (in.)

t_s, t_{s_1}, t_{s_2} - stiffener thickness (in.)

u, v, w - components of the displacement vector (in.)

u_o, v_o, w - components of the displacement vector at the midsurface of the laminate (in.)

V - (1) shear force (lb)

(2) sandwich shear stiffness parameter $\left(\frac{\pi}{b}\right)^2 \frac{\sqrt{D_{11}D_{22}}}{DQ_x}$

α - (1) coefficient of thermal expansion (in./in./°F)

- (2) the quantity $\sqrt{E_x/E_y}$ (used in section IV only)

α_L - lamina coefficient of thermal expansion along L axis (in./in./°F)

2, 3

3

α_T - lamina coefficient of thermal expansion along T axis (in./in./°F)

α_x^c - laminate coefficient of thermal expansion along X axis (in./in./°F)

α_y^c - laminate coefficient of thermal expansion along Y axis (in./in./°F)

α_{xy}^c - laminate shear distortion coefficient of thermal expansion (in./in./°F)

β - the quantity $\alpha\nu_{yx} + 2\gamma$ (used in section IV only)

γ - (1) shear strain (in./in.)

- (2) ratio $\dfrac{D_{66}}{\sqrt{D_{11}D_{22}}}$

Δ - difference (used as a prefix to quantitative symbols)

ϵ - strain (in./in.)

η - honeycomb core shear modulus ratio $\dfrac{G'_{cy}}{G'_{cx}}$

θ - angular orientation of a lamina in a laminate, i.e., the angle between L and X axes (°)

λ_{xy} - product of $\nu_{xy}\nu_{yx}$

ν - Poisson's ratio

ν_{LT} - Poisson's ratio relating contraction in the T direction due to extension in the L direction

ν_{TL} - Poisson's ratio relating contraction in the L direction due to extension in the T direction

ν_{xy} - Poisson's ratio relating contraction in the y direction due to extension in the x direction

ν_{yx} - Poisson's ratio relating contraction in the x direction due to extension in the y direction

ρ - density (lb/ft^3)

ρ'_c - honeycomb sandwich core density (lb/ft^3)

ρ_f - density of honeycomb sandwich face sheets (lb/ft^3)

σ - applied normal stress ($lb/in.^2$)

τ - applied shear stress ($lb/in.^2$)

σ_{ij}, τ_{ij} - stress in j direction on surface whose outer normal is in i direction (i,j = 1,2,3 or x,y,z) ($lb/in.^2$)

ϕ - ratio $\frac{N_y}{N_x}$

ψ - the quantity $\frac{\sqrt{E_x E_y}}{1 - \nu_{xy}\nu_{yx}}$

SUBSCRIPTS AND SUPERSCRIPTS

c - composite laminate (as distinguished from separate laminae)

f - face sheet

L, T, Z - laminae natural orthogonal coordinates

s - stiffener

X, Y, Z - general coordinate system, laminate coordinate system

Section I

INTRODUCTION

The purpose of this program was to take the first step toward the generation and presentation of basic engineering data necessary to perform high-confidence-level structural design of primary aircraft structures utilizing advanced composite materials. The program was limited to an in-depth generation of basic material allowables for one boron/epoxy and several graphite/epoxy material systems, and the determination of basic structural element response for the boron/epoxy system alone. The boron portion of this program was conducted in conjunction with a concurrent General Dynamics/Forth Worth (GD/FW) program which was funded under Air Force Contract F33615-68-C-1474. The boron/epoxy material system highlighted by both these programs was Narmco 5505, furnished by the supplier as 3-inch prepreg tape. This is a composite material consisting of collimated 4-mil boron filaments, 208 per inch of tape width, embedded in a matrix of Narmco 2387 epoxy resin, and supported on a 1-mil layer of 104 glass scrim cloth. The graphite portion of this program is being conducted independent of any other program, and will be described in fuller detail in a later volume of this report. Additional data for all these materials, as well as for other filament/matrix material systems, were obtained from published Government, industry, and technical journal reports, and were used to augment the data generated in this program.

This program was composed of three major work task areas:

Task I - Generation of Composite Material Design Allowables

Task II - Structural Element Test Program and Analysis Evaluation

Task III - Development of Advanced Composite Structural Design Manual for Aircraft

Task I is divided into two distinct areas of effort by the separate boron/epoxy and graphite/epoxy programs. The purpose of the boron portion of task I was to complement the basic material design allowable activities conducted by GD/FW (reference 12) and to develop acceptable laminate fabrication and inspection procedures. The boron effort was divided into the following work areas: The establishment of program coordination procedures for the North American Rockwell Corporation and General Dynamics related programs; the accomplishment of a limited material development program; the generation of basic allowables for the constituent materials; establishment of the accuracy of current analytical procedures for predicting certain basic allowables; and the development, where reliable techniques were lacking, of prediction techniques for these basic material allowables.

The graphite portion of task I consists primarily of a screening and characterization of several graphite/epoxy material systems and will be delineated in a later volume of this report.

The purpose of task II, which was concerned solely with boron/epoxy material, was to generate data on basic structural elements which form the building blocks from which aircraft structures are designed. A minimum evaluation of structural elements was conducted, including one basic laminate and one elevated temperature. Factors which were considered in the detail design of the structural elements included laminate orientations, panel proportions and edge restraints, effectiveness of typical forms of panel stabilization, evaluation of cutouts, and thermal gradient effects. One or more elements were selected for each primary and/or combined load applications. The test program included local and general instability of flat panels and natural frequency determinations. The results of this test program were compared to predicted response, failure mode, and strength techniques for basic structural elements.

The task III work area was originally centered on the development of an advanced composite structural design manual for aircraft structures. The first effort of this task involved revision and refinement of the Aircraft Division of the Intermediate Draft of the Structural Design Guide developed by the Southwest Research Institute, San Antonio, Texas, under Air Force Contract AF33(615)-5142. The completely revised and reorganized Aircraft Division resulting from this phase of effort was published in the Final Draft of the Design Guide in November 1968 under Contract AF33(615)-68-C-1241. Soon thereafter, a review of the Final Draft by a select industry group led to a decision by AFML to reorganize the entire Design Guide for the First Edition, which was then assigned to NR/LAD under Contract F33615-69-C-1368.

Subsequent phases of task III of this program, in light of the foregoing developments, consisted of the preparation of the Aircraft System Applications chapter of the First Edition of the Design Guide as well as the preparation of data generated by tasks I and II of this program for incorporation into the various technical function-oriented chapters. Task III also included the incorporation into the Design Guide of data generated by the concurrent GD program.

The bulk of the basic material allowables for the 5505 material system was generated by the General Dynamics contract. This concurrent and integrally related contract was coordinated with the Los Angeles Division program effort through scheduled periodic coordination meetings. These meetings insured the continuous flow of pertinent program data between the two contractors.

This report is divided into four separate volumes, in each of which the subject areas of interest comprise an independent segment of the overall program. Each volume is a self-contained document, complementing the other three

volumes but not dependent upon them for coherence or continuity. The titles of the four volumes are:

Volume I - Material and Basic Allowable Development - Boron/Epoxy

Volume II - Structural Element Behavior - Test and Analytical Determination

Volume III - Theoretical Methods

Volume IV - Material and Basic Allowable Development - Graphite/Epoxy

Volume II contains the analytical derivation of the behavioral equations and relationships characteristic of anisotropic composites. It is divided into a number of independent sections, each of which treats a particular structural element concept or some separate facet of advanced composites application to structural design.

Section II is fundamental in scope and concerns the extensional and flexural elastic constants of filamentary composite laminae and laminates. Sections III, IV, and V cover the behavioral characteristics of flat plates, orthotropic sandwich plates and shells, and stiffened skin construction, respectively. Section IV discusses the general approach to minimum weight design, with detailed attention to zee- and hat-stiffened elements, while Section VII is devoted to the thermoelastic relationships of composites. Section VIII concerns the special topic of composite panel joints, and includes separate treatment of several types of joint.

Laminate ply orientations are described and specified in this report by use of the laminate orientation code defined in the Structural Design Guide for Advanced Composite Applications.

SECTION II

EXTENSIONAL AND FLEXURAL ELASTIC CONSTANTS

STRESS-STRAIN RELATIONS OF A LAMINA IN GENERALIZED PLANE STRESS

Consider the fiber reinforcement to be unidirectional. Referring to a set of orthogonal axes, L, T, and Z, with L parallel to the fiber direction and Z = 0 coinciding with the middle surface of the lamina, then the stress-strain relation of the lamina shown in figure 1 may be represented by

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \frac{\gamma_{LT}}{2} \end{Bmatrix} \tag{1}$$

where σ_L, σ_T are direct stresses, σ_{LT} is the shear stress. ϵ_L, ϵ_T are direct strains, and γ_{LT} is the shear strain.

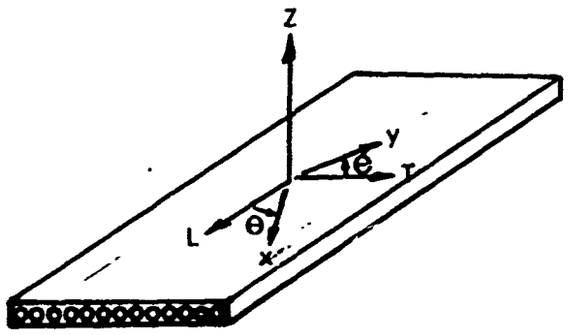


Figure 1. Typical Lamina

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}} \quad , \quad Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{12} = Q_{21} = \frac{E_T \nu_{LT}}{1 - \nu_{LT}\nu_{TL}} = \frac{E_L \nu_{TL}}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{66} = G_{LT}$$

E_L, E_T denote the Young's modulus in L and T directions, respectively, G_{LT} denotes the shear modulus, and ν_{LT} and ν_{TL} denote the major and minor Poisson's ratios, respectively.

Let X, Y, and Z be a set of rectangular axes where the X and Y axes are obtained by rotating L and T axes through an angle θ as shown in figure 2.

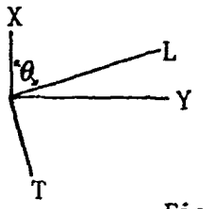


Figure 2. Lamina and Laminate Axes

The stress-strain relation referring to X, Y axes is obtained by transforming equation 1 as

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_{XY} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_X \\ \epsilon_Y \\ \gamma_{XY} \end{Bmatrix} \tag{2}$$

where

$$\bar{Q}_{11} = Q_{11}\, m^4 + 2(Q_{12} + 2Q_{66})\, m^2 n^2 + Q_{22}\, n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\, m^2 n^2 + Q_{12}(m^4 + n^4) = \bar{Q}_{21}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\, m^3 n + (Q_{12} - Q_{22} + 2Q_{66})\, mn^3 = \bar{Q}_{61}$$

$$\bar{Q}_{22} = Q_{11}\, n^4 + 2(Q_{12} + 2Q_{66})\, m^2 n^2 + Q_{22}\, m^4$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\, mn^3 + (Q_{12} - Q_{22} + 2Q_{66})\, m^3 n = \bar{Q}_{62}$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\, m^2 n^2 + Q_{66}\,(m^4 + n^4)$$

$$m = \cos\theta$$

$$n = \sin\theta$$

It should be noted that the stress-strain relation referring to the lamina axes (equation 1) is orthotropic, i.e., no coupling between the extensional and shearing stresses and strains, while the stress-strain relation referring to the laminate axes (equation 2) is generally not orthotropic.

SECTIONAL FORCES AND MOMENTS OF A LAMINATE IN TERMS OF DISPLACEMENTS

From Kirchhoff's assumption, normals to the middle plane of the laminate remain normal to the middle surface of the laminate during deformation, and the normal stress on planes parallel to the middle surface may be neglected in comparison with other stress components. Let u, v and w be the displacement along the X, Y and Z axes, and let u_0, v_0 and w_0 be the corresponding displacement of points on the middle plane. We have

$$\begin{aligned} u &= u_0 - z\frac{\partial w}{\partial x} \\ v &= v_0 - z\frac{\partial w}{\partial y} \\ w &= w_0 \end{aligned} \tag{3}$$

For small strain,

$$\epsilon_x = \frac{\partial u}{\partial x},\ \epsilon_y = \frac{\partial v}{\partial y},\ \gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{4}$$

Substituting equation 4 into equation 2, we have

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = \left[\bar{Q}_{ij}\right]_i \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} - z\dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial v_0}{\partial y} - z\dfrac{\partial^2 w_0}{\partial y^2} \\ \left(\dfrac{\partial v_0}{\partial x} + \dfrac{\partial u_0}{\partial y}\right) - 2z\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{5}$$

where the subscript "i" denotes the i^{th} lamina in the laminate. The notations of the laminate are shown in figure 3. For a laminate of N laminae, the section forces and moments shown in figure 4 are defined as

$$\left\{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}\right\} =$$

$$\sum_{i=1}^{N} \int_{h_i}^{h_{i+1}} \left(\sigma_x^i, \sigma_y^i, \sigma_{xy}^i, z\sigma_x^i, z\sigma_y^i, z\sigma_{xy}^i\right) dz \qquad (6)$$

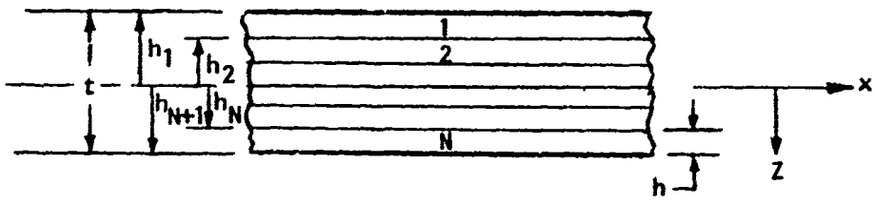


Figure 3. Laminate of N Laminae

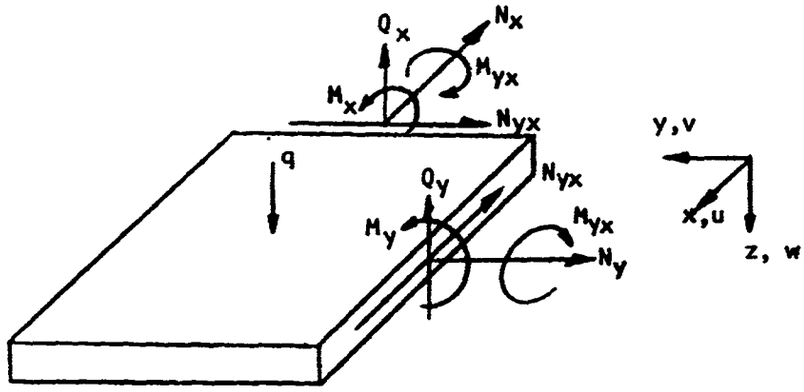


Figure 4. Positive Direction of Internal Forces and Moments

The sign conventions for those section forces and moments are shown in figure 4. The laminae are now considered to be symmetrically laid with respect to the middle plane of the laminate. Performing the integration as shown in equation 6, we have

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{ij} \end{bmatrix} \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ (u_{0,y} + v_{0,x}) \end{Bmatrix} \tag{7a}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = - \begin{bmatrix} D_{ij} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{2\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{7b}$$

LAMINATE EXTENSIONAL CONSTANTS

Since all the laminae are of the same thickness, we have from figure 3

$$h = \frac{t}{N} \tag{8a}$$

$$h_i = (i-1)\, h - \frac{t}{2}$$

$$h_{i+1}^3 - h_i^3 = \frac{h^3}{3}\left[(3i^2-3i+1) + \frac{3N}{2}(1-2i) + \frac{3N^2}{4}\right] \quad \text{for } i=1, 2, \ldots, N \tag{8b}$$

$$= \frac{1}{3}\left(\frac{t}{N}\right)^3 \left[3\left(i - \frac{N+1}{2}\right)^2 + \frac{1}{4}\right]$$

and

$$\begin{Bmatrix} A_{ij} \\ D_{ij} \end{Bmatrix} = \sum_{i=1}^{N} \bar{Q}_{ij}^{(i)} \begin{Bmatrix} h \\ \frac{h^3}{3}\left[3\left(i-\frac{N+1}{2}\right)^2 + \frac{1}{4}\right] \end{Bmatrix} \tag{9}$$

Let us now further consider only the cases where the number of laminae with $+\theta$ is equal to that with $-\theta$, i.e., the layup is balanced. Then

$$A_{16} = A_{61} = A_{26} = A_{62} = 0 \tag{10}$$

and equation 7a reduces to

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{11}$$

The stress-strain relation corresponding to equation 11 for a homogeneous plate is given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{1}{t} \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{12}$$

Comparing this with equation 1, it is seen that the equivalent homogeneous elastic constants of the laminate are then

$$\begin{aligned} \nu_{yx} &= A_{12}/A_{11} \\ \nu_{xy} &= A_{12}/A_{22} \\ E_x &= \frac{A_{11}A_{22} - A_{12}^2}{t\,A_{22}} \\ E_y &= \frac{A_{11}A_{22} - A_{12}^2}{t\,A_{11}} \\ G_{xy} &= \frac{A_{66}}{t} \end{aligned} \tag{13}$$

In addition, since the laminate is considered to be laid up symmetrically relative to its midsurface, equation 9 reduces to

$$\begin{Bmatrix} A_{ij} \\ D_{ij} \end{Bmatrix} = \sum_{i=\frac{N}{2}+1}^{N} \begin{Bmatrix} 2h \\ \frac{2h^3}{3}\left[3\left(i - \frac{N+1}{2}\right)^2 + \frac{1}{4}\right] \end{Bmatrix} \bar{Q}_{ij}^{(i)} \tag{14}$$

D_{ij}'s OF SOME COMMONLY USED LAMINATES

Let us now consider symmetric laminates composed of q sets of plies with each set having p_1 plies at 0°, $2p_\theta$ plies at $\pm\theta$°, and p_2 plies at 90° as shown in figure 5.

Let

$$\Phi(i) = 3\left(i - \frac{N+1}{2}\right)^2 + \frac{1}{4} \tag{15}$$

and

$$r = p_1 + 2p_\theta + p_2$$

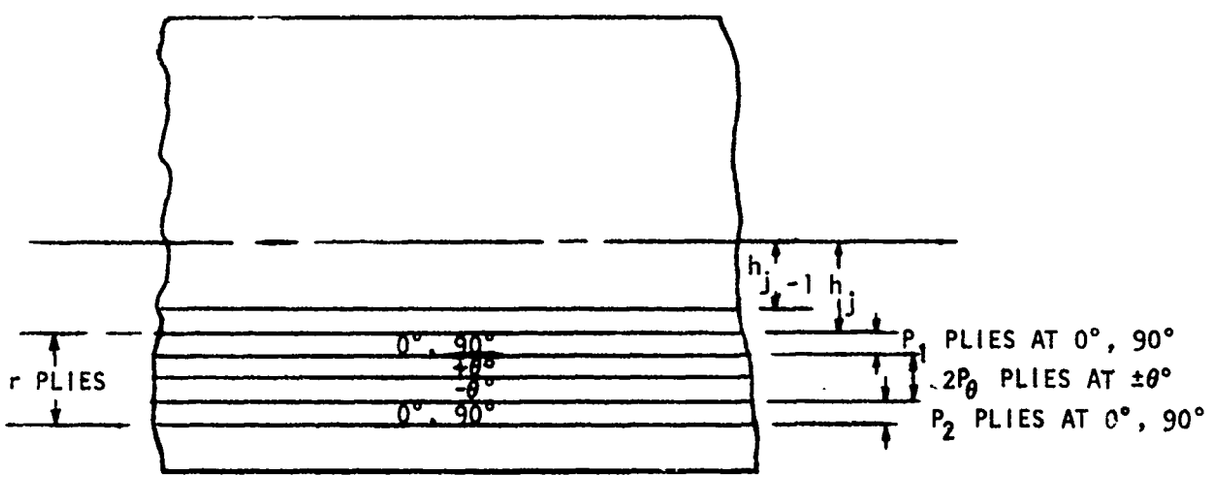


Figure 5. Typical Symmetrical Laminate

then

$$D_{11} = \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11}\right)_{0^\circ} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)\,r}^{\left(\frac{N}{2}+1\right)+(k-1)\,r+p_1-1} \Phi(i) \right.$$

$$+ \left(\bar{Q}_{11}\right)_{\theta^\circ} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)\,r+p_1+2p_\theta-1}^{\left(\frac{N}{2}+1\right)+(k-1)\,r+p_1+2p_\theta-1} \Phi(i)$$

$$\left. + \left(\bar{Q}_{11}\right)_{90^\circ} \sum_{i=\left(\frac{N}{2}+1\right)+(k-1)\,r+p_1+2p_\theta}^{\left(\frac{N}{2}+1\right)+kr-1} \Phi(i) \right\}$$

$$D_{11} = \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11}\right)_{0^\circ} \sum_{j=1}^{p_1} \Phi\left[\frac{N}{2} + j + (k-1)\,r\right] \right.$$

$$+ \left(\bar{Q}_{11}\right)_{\theta^\circ} \sum_{j=1}^{2p_\theta} \Phi\left[\frac{N}{2} + j + (k-1)\,r + p_1\right]$$

$$\left. + \left(\bar{Q}_{11}\right)_{90^\circ} \sum_{j=1}^{p_2} \Phi\left[\frac{N}{2} + j + (k-1)\,r + p_1 + 2p_\theta\right] \right\} \qquad (16)$$

$$= \frac{2}{3} h^3 \sum_{k=1}^{q/2} \left\{ \left(\bar{Q}_{11}\right)_{0^\circ} \sum_{j=1}^{p_1} \Phi\left(j + kr + r_1\right) \right.$$

$$\left. + \left(\bar{Q}_{11}\right)_{\theta^\circ} \sum_{j=1}^{p_{2_\theta}} \Phi\left(j + kr + r_2\right) + \left(\bar{Q}_{11}\right)_{90^\circ} \sum_{j=1}^{p_2} \Phi\left(j + kr + r_3\right) \right\}$$

where

$$r_1 = \frac{N}{2} - r$$

$$r_2 = \frac{N}{2} - 2p_\theta - p_2 \tag{17}$$

$$r_3 = \frac{N}{2} - p_2$$

Letting

$$C_A = \sum_{k=1}^{q/2} \sum_{j=1}^{p_1} \boldsymbol{\Phi}\,(j + kr + r_1)$$

$$C_B = \sum_{k=1}^{q/2} \sum_{j=1}^{2p_\theta} \boldsymbol{\Phi}\,(j + kr + r_2) \tag{18}$$

$$C_C = \sum_{k=1}^{q/2} \sum_{j=1}^{p_2} \boldsymbol{\Phi}\,(j + kr + r_3)$$

then

$$D_{11} = \frac{2}{3} h^3 \left[C_A \left(\bar{Q}_{11}\right)_0 + C_B \left(\bar{Q}_{11}\right)_\theta + C_C \left(\bar{Q}_{11}\right)_{90} \right] \tag{19}$$

Expanding $\boldsymbol{\Phi}$ in equation 18 and noting that

$$\sum_{n=1}^{N} n^2 = \frac{N}{6}\,(N+1)\,(2N+1)$$

$$\tag{20}$$

$$\sum_{n=1}^{N} n = \frac{N}{2}\,(N+1)$$

$$C_A = \frac{q\,p_1}{4}\left\{(p_1+1)\left[2p_1 + 1 + 3\,(2r_1 - N - 1)\right] + \frac{3}{2}\,(2r_1 - N - 1)^2 + \frac{1}{2}\right\}$$

$$+ 3rp_1\,(p_1 + 2r_1 - N)\,\frac{q}{2}\,\frac{\left(\frac{q}{2}+1\right)}{2} + 3p_1\,r^2\,\frac{q}{2}\,\frac{\left(\frac{q}{2}+1\right)\left(q+1\right)}{6} \tag{21}$$

$$= \frac{q\,p_1}{8}\Big[2(p_1+1)\,(2p_1 + 6r_1 - 3N - 2) + 3\,(2r_1 - N - 1)^2$$

$$+ r\,(q+2)\,(3p_1 + 6r_1 - 3N + r + qr) + 1\Big]$$

By replacing r_1 by r_2 and p_1 by $2p_\theta$ in the preceding equation, we obtain

$$C_B = \frac{qp_\theta}{4}\Big[2(2p_\theta+1)\,(4p_\theta+6r_2-3N-2) + 3(2r_2-N-1)^2$$

$$+ r(q+2)\,(6p_\theta+6r_2-3N+r+qr) + 1\Big] \tag{22}$$

By replacing r_1 by r_3 and p_1 by p_2 in equation 21, we obtain

$$C_C = \frac{qp_2}{8}\Big[2(p_2+1)\,(2p_2+6r_3-3N-2) + 3(2r_3-N-1)^2$$

$$+ r(q+2)\,(3p_2+6r_3-3N+r+qr) + 1\Big] \tag{23}$$

The preceding results are also applicable to D_{12}, D_{16}, D_{22}, D_{26}, and D_{66}. The resulting expressions are

$$(D_{11}, D_{12}, D_{16}, D_{22}, D_{26}, D_{66}) = \frac{2}{3}h^3\Big\{\Big[(\bar{Q}_{11})_0, (\bar{Q}_{12})_0, 0, (\bar{Q}_{22})_0, 0, (\bar{Q}_{66})_0\Big]C_A$$

$$+ \Big[(\bar{Q}_{11})_\theta, (\bar{Q}_{12})_\theta, (\bar{Q}_{16})_\theta, (\bar{Q}_{22})_\theta, (\bar{Q}_{26})_\theta, (\bar{Q}_{66})_\theta\Big]C_B \tag{24}$$

$$+ \Big[(\bar{Q}_{11})_{90}, (\bar{Q}_{12})_{90}, 0, (\bar{Q}_{22})_{90}, 0, (\bar{Q}_{66})_{90}\Big]C_C\Big\}$$

Equation 24 represents the closed-form sums for the flexural stiffnesses of a laminate composed of q repeated sets of $[0_{p_1}/\pm\theta_{p_\theta}/90_{p_2}]$ laid up

symmetrically relative to the midsurface of the laminate. These stiffnesses are required to conduct analyses of orthotropic laminated plates. The equivalent extensional constants are obtained by combining equations 13 and 14 in the appropriate manner.

For cases when a more general laminate layup is under consideration, equation 9, which is valid for any laminate ply orientations, should be utilized.

CALCULATIONS OF EXTENSIONAL AND FLEXURAL CONSTANTS

A computer program, AC-2, has been developed to perform the calculations required to determine the extensional elastic constants and flexural stiffnesses for a balanced laminate laid up symmetrically relative to its midsurface. This program incorporates the closed-form solution for D_{ij}'s described by equation 24 and, as such, is very rapid for thick as well as thin laminates. The elastic constants, as defined by equation 13, for several laminate families of Narmco 5505 as obtained from AC-2 are plotted in figures 6 through 17 for room temperature (RT) and 350° F. The elastic constants of each lamina in its natural coordinate system (L, T, Z) were obtained from reference 2.

The elastic constants that can be found from these curves are E_x, E_y, G_{xy}, ν_{xy}, and ν_{yx}. To assist the designer further in choosing the desired laminate orientation, lines were drawn which represent both "% at 0°" and "% at 90°," even though only one need be given to derive the other.

Also, it should be noted that the G_{xy} curves are independent of the "% at 0°" versus the "% at 90°" plies. That is, as long as the "% at ±θ°" values are known, then the remaining relative percentage of the 0° versus the 90° plies does not affect G_{xy}. Furthermore, considering the number of plies at 0°, ±θ°, and 90° to be n1, n2, and n3, respectively, then it is seen that

1. No curve is given for E_y for $[0_{n1}/\pm 45_{n2}/90_{n3}]$ laminate orientations. This is because E_y for these orientations can be found by using the curves for E_x after interchanging the "% at 0°" and "% at 90°" labels.

2. The curves for $[0_{n1}/\pm 30_{n2}/90_{n3}]$ laminate orientations are not given, but the curves for the $[0_{n1}/\pm 60_{n2}/90_{n3}]$ laminate orientations can be used for $[0_{n1}/\pm 30_{n2}/90_{n3}]$ properties if the following are considered:

   a. E_x for $[0_{n1}/\pm 30_{n2}/90_{n3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and using the E_y curves for $[0_{n1}/\pm 60_{n2}/90_{n3}]$.

b. E_y for $[0_{n1}/\pm30_{n2}/90_{n3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and using the E_x curves for $[0_{n1}/\pm60_{n2}/90_{n3}]$.

c. ν_{xy} for $[0_{n1}/\pm30_{n2}/90_{n3}]$ is found by interchanging the "% at 0°" and "% at 90°" labels and multiplying the ν_{xy} value from the $[0_{n1}/\pm60_{n2}/90_{n3}]$ curves by the ratio E_x/E_y for $\left[0_{n1}/\pm30_{n2}/90_{n3}\right]$.

The program also computes the flexural stiffnesses of the laminate assuming it to be a homogeneous orthotropic medium and thus utilizing the extensional constants, e.g., $D_{11} = E_x\, t^3/12(1-\nu_{xy}\,\nu_{yx})$, and compares these stiffnesses to those obtained by the rigorous method, i.e., equation 14. This output permits both the determination of how thick a laminate of a certain orientation must be before the stiffnesses D_{16} and D_{26} approach zero, and determination of the error associated with the use of flexural stiffnesses D_{11}, D_{12}, D_{22}, and D_{66} based on the extensional constants rather than equation 14.

The latter data permit an assessment of when the classical solutions for the response behavior of homogeneous orthotropic plate and shell elements are valid (for the orientation in question), i.e., cases where the ratios D_{16}/D_{11} and D_{26}/D_{11} approach zero. A further discussion of the significance of the stiffness terms D_{16} and D_{26} and the ratios D_{16}/D_{11} and D_{26}/D_{11} in obtaining static and dynamic response solutions to solid laminated plates is contained in section III.

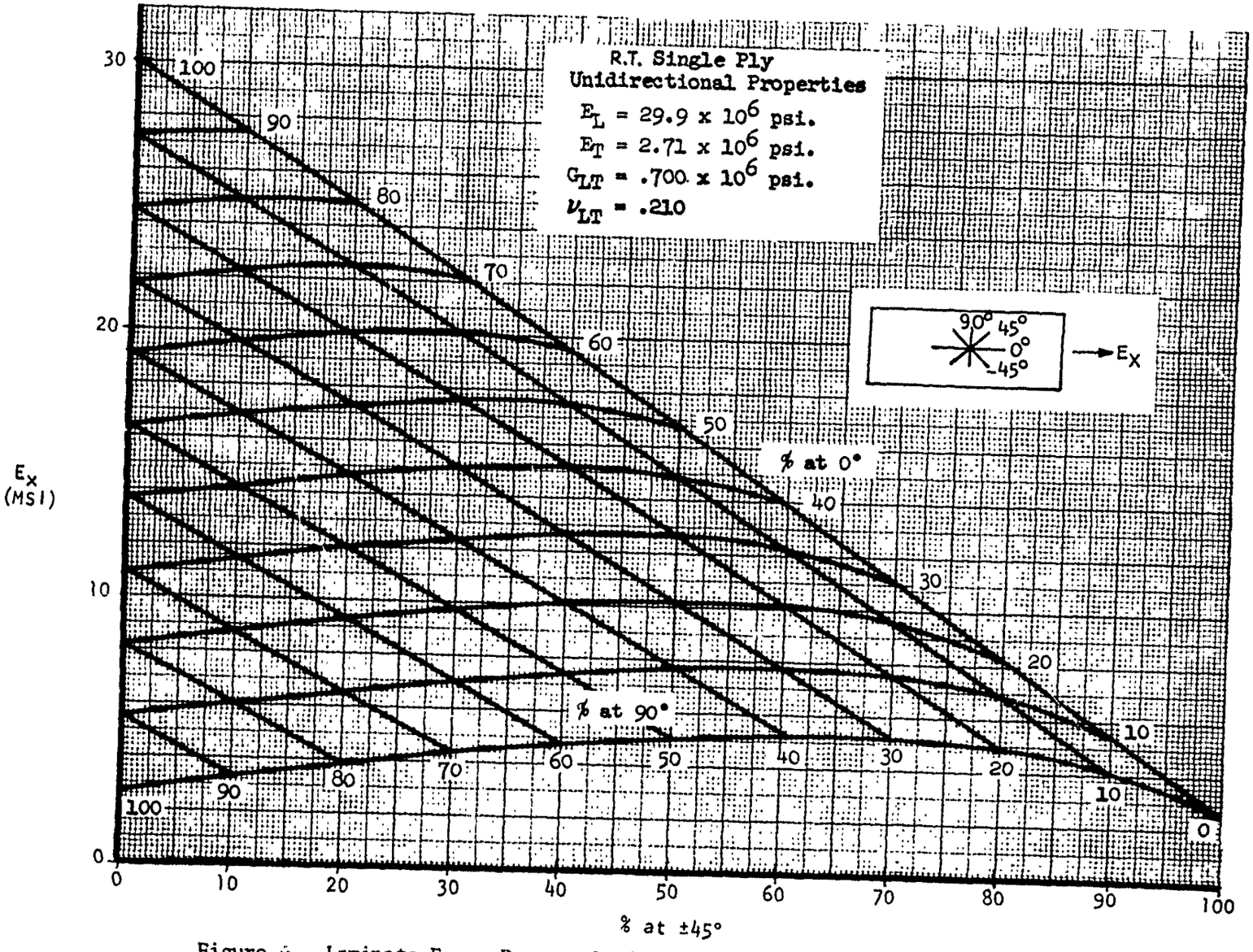


Figure 6. Laminate E_x vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT

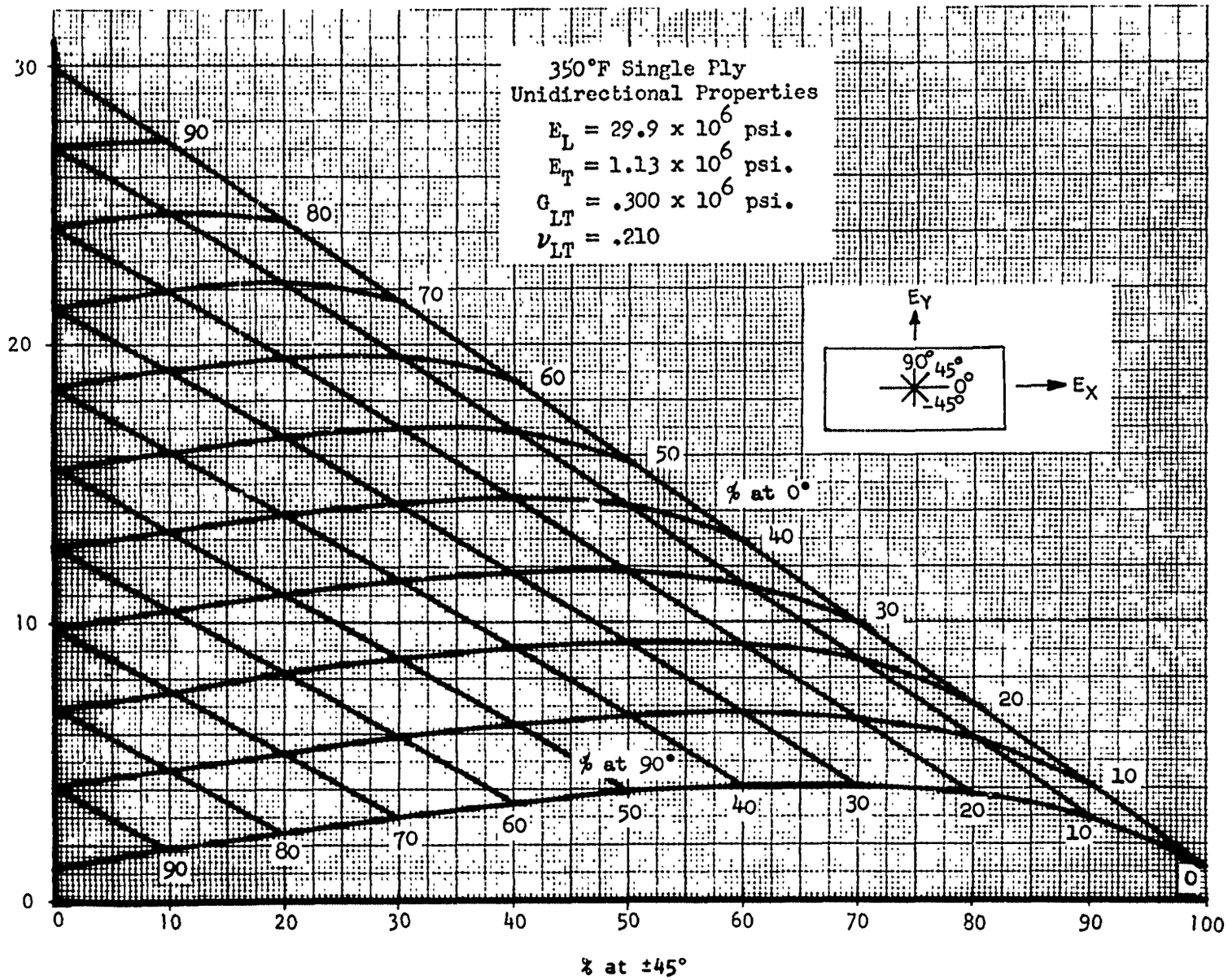


Figure 7. Laminate E_x vs Percent Laminae at o°, ±45°, and 90° for Narmco 5505 at 350° F

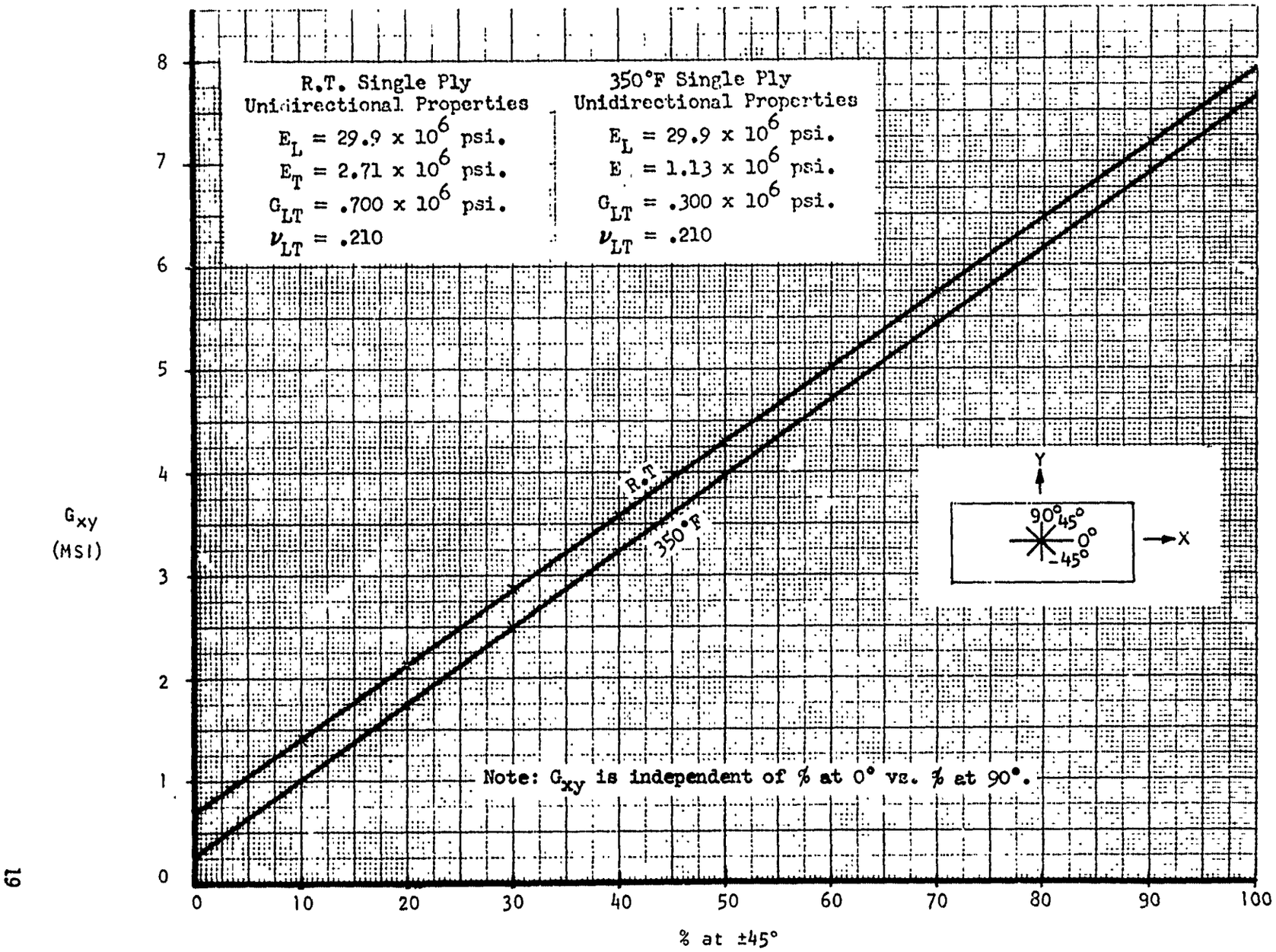


Figure 8. Laminate G_{XY} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT and 350° F

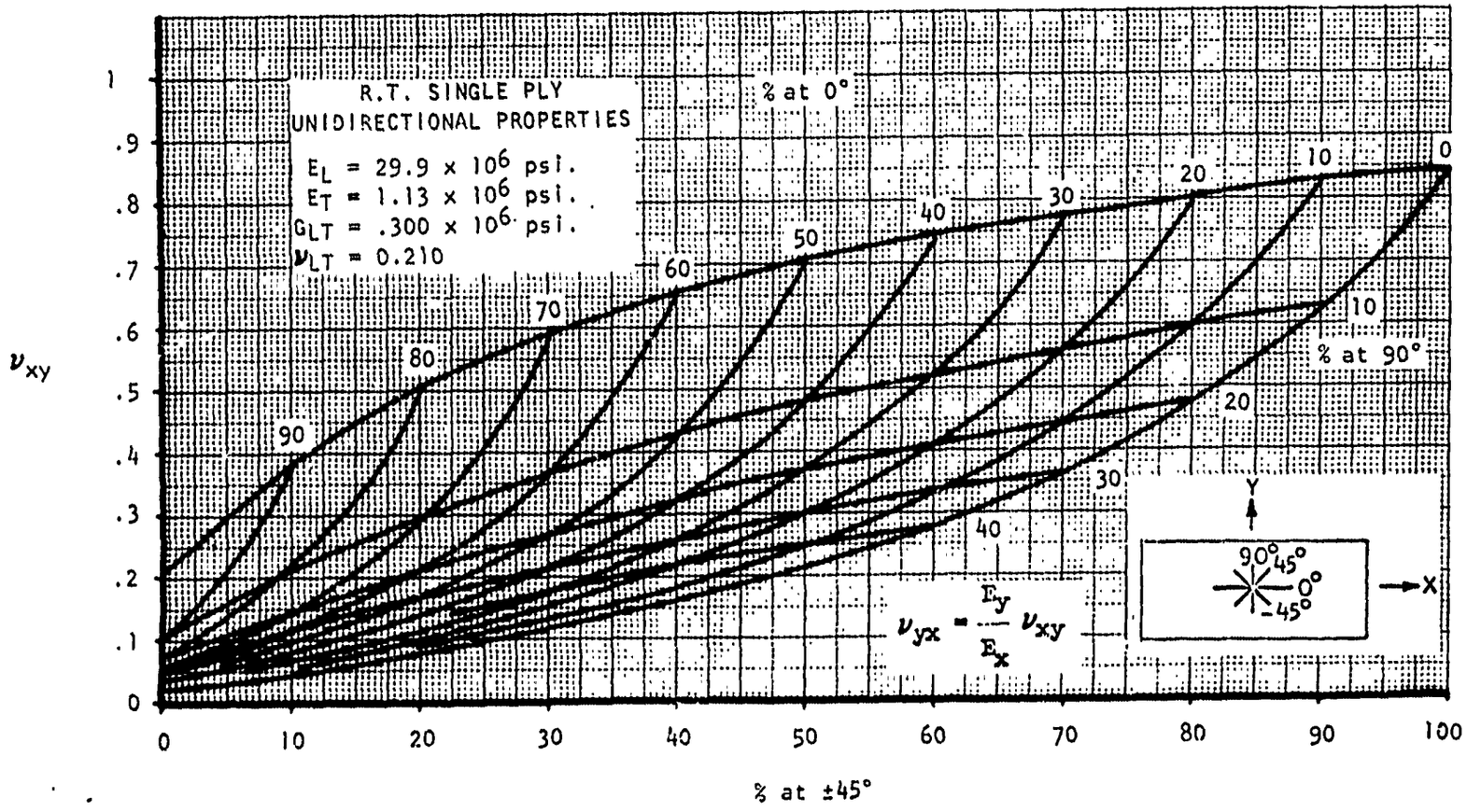


Figure 9. Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at RT

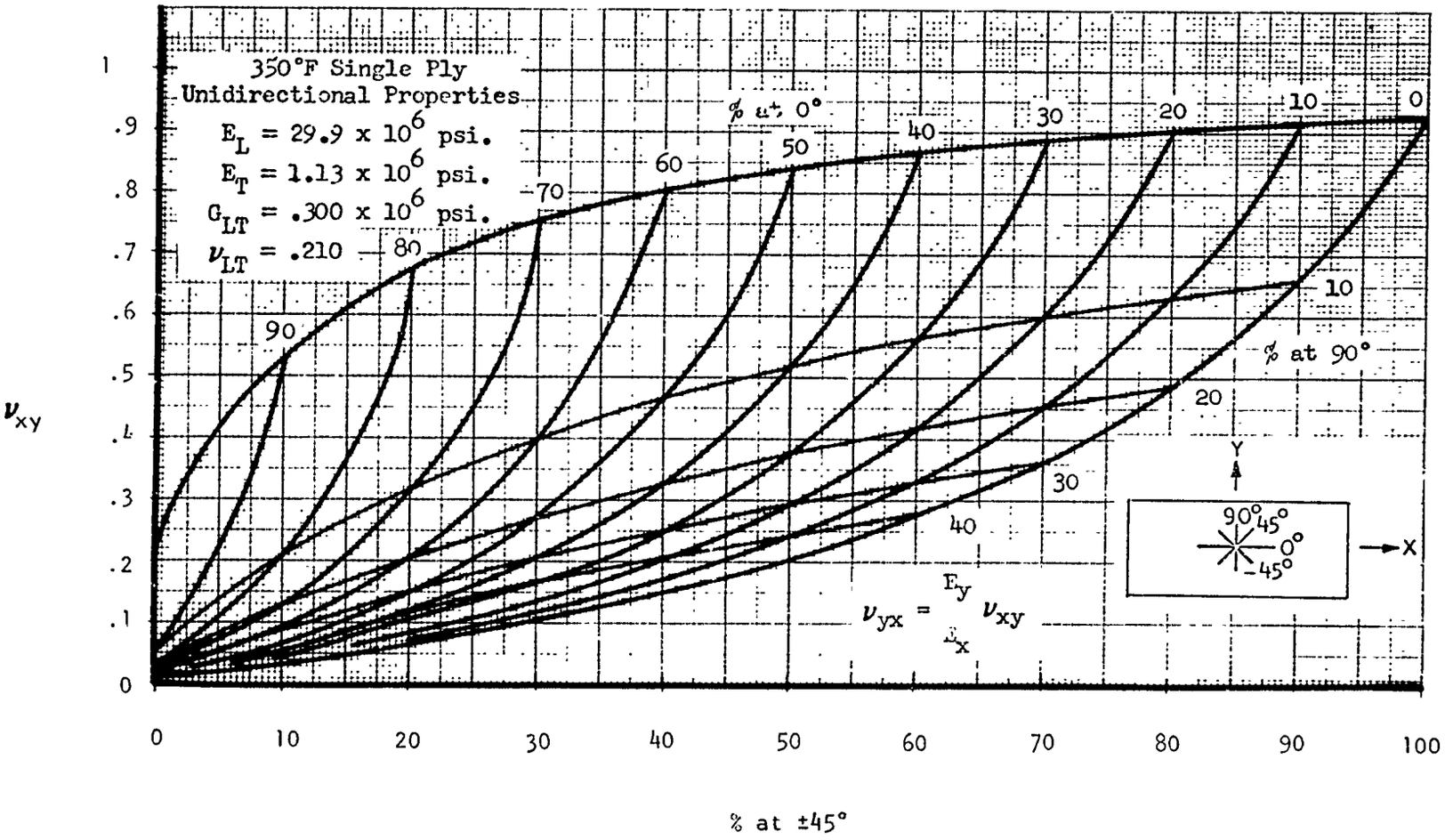


Figure 10. Laminate ν_{xy} vs Percent Laminae at 0°, ±45°, and 90° for Narmco 5505 at 350° F

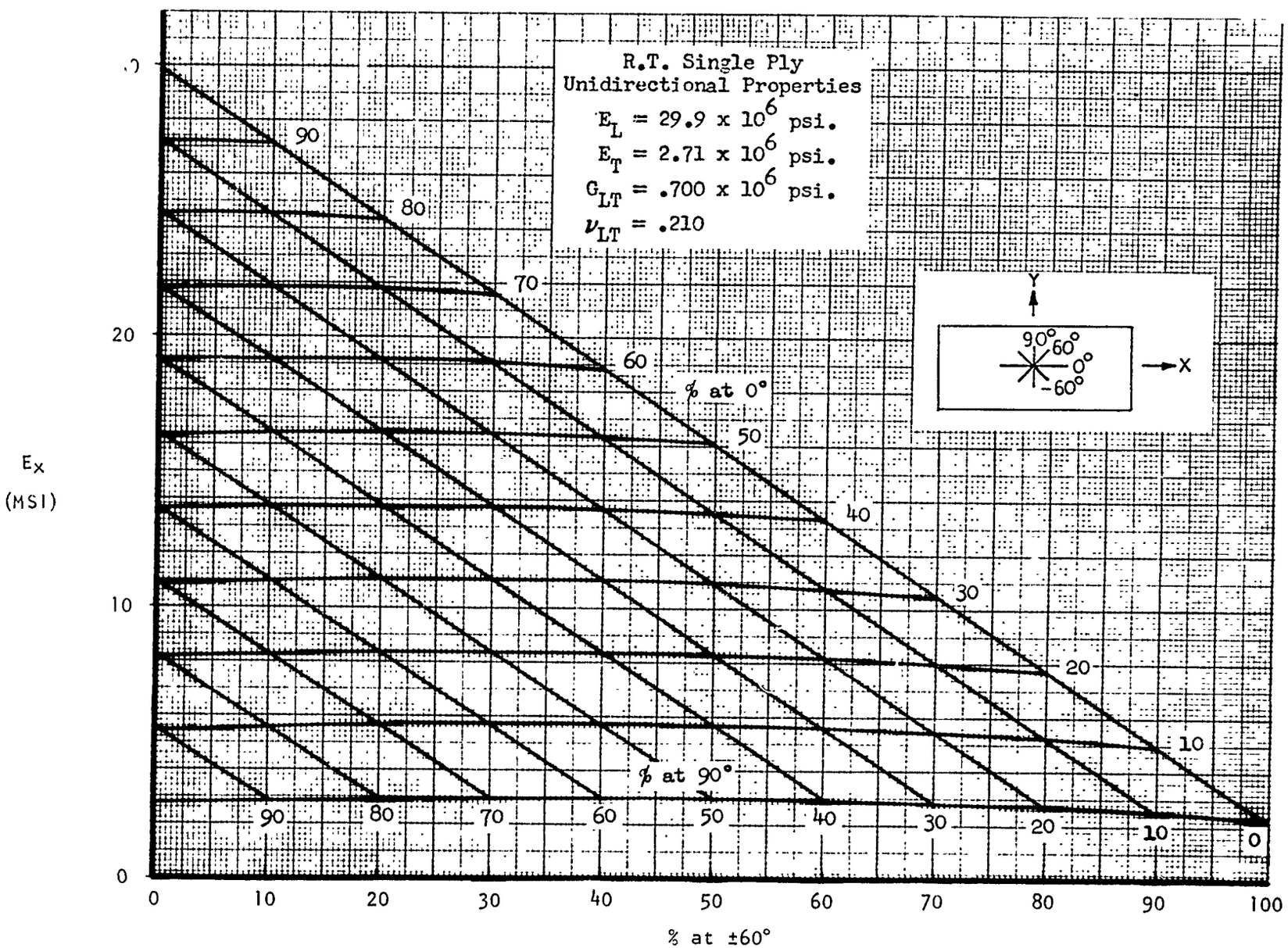


Figure 11. Laminate E_X vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT

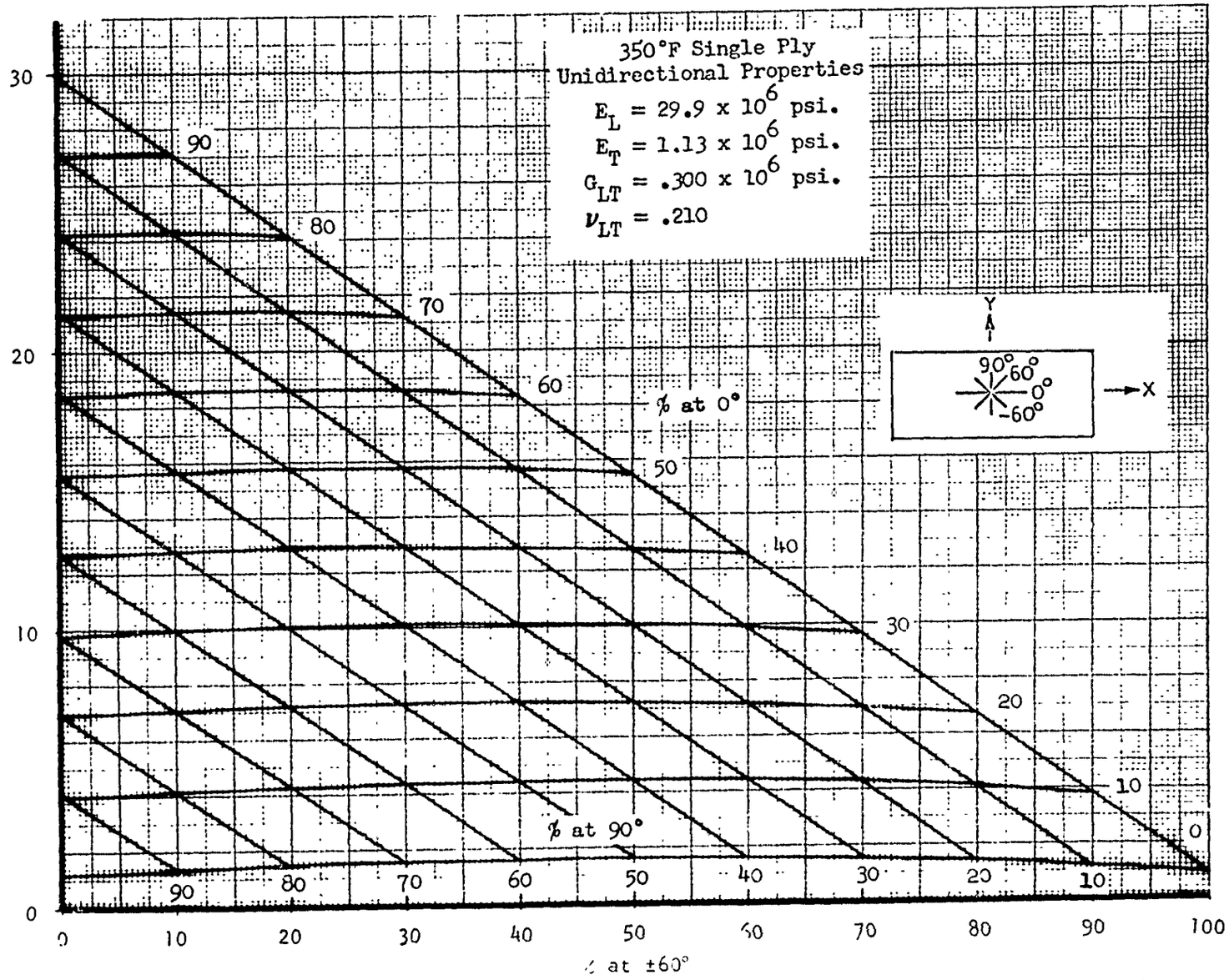


Figure 12. Laminate E_x vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F

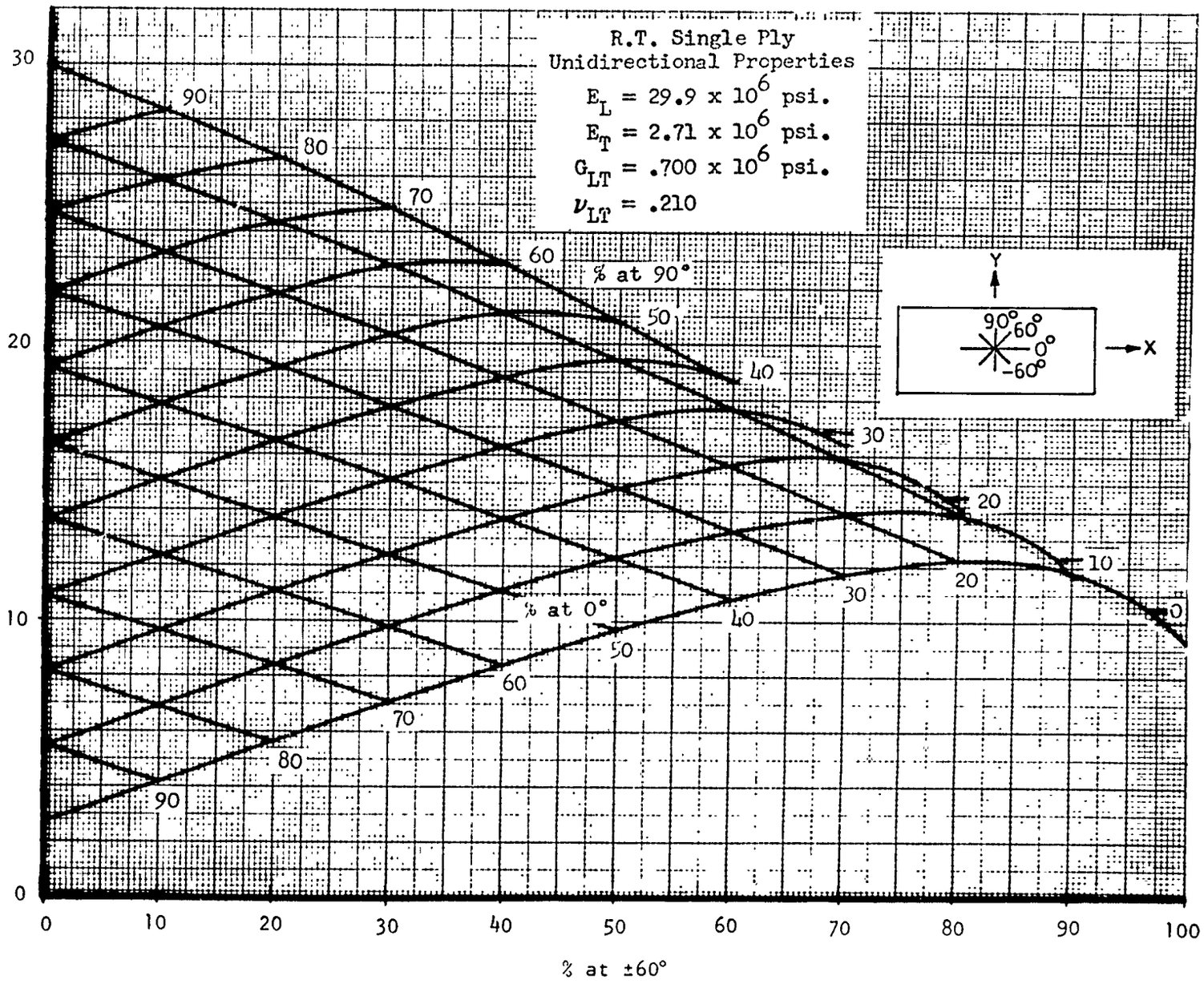


Figure 13. Laminate E_y vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT

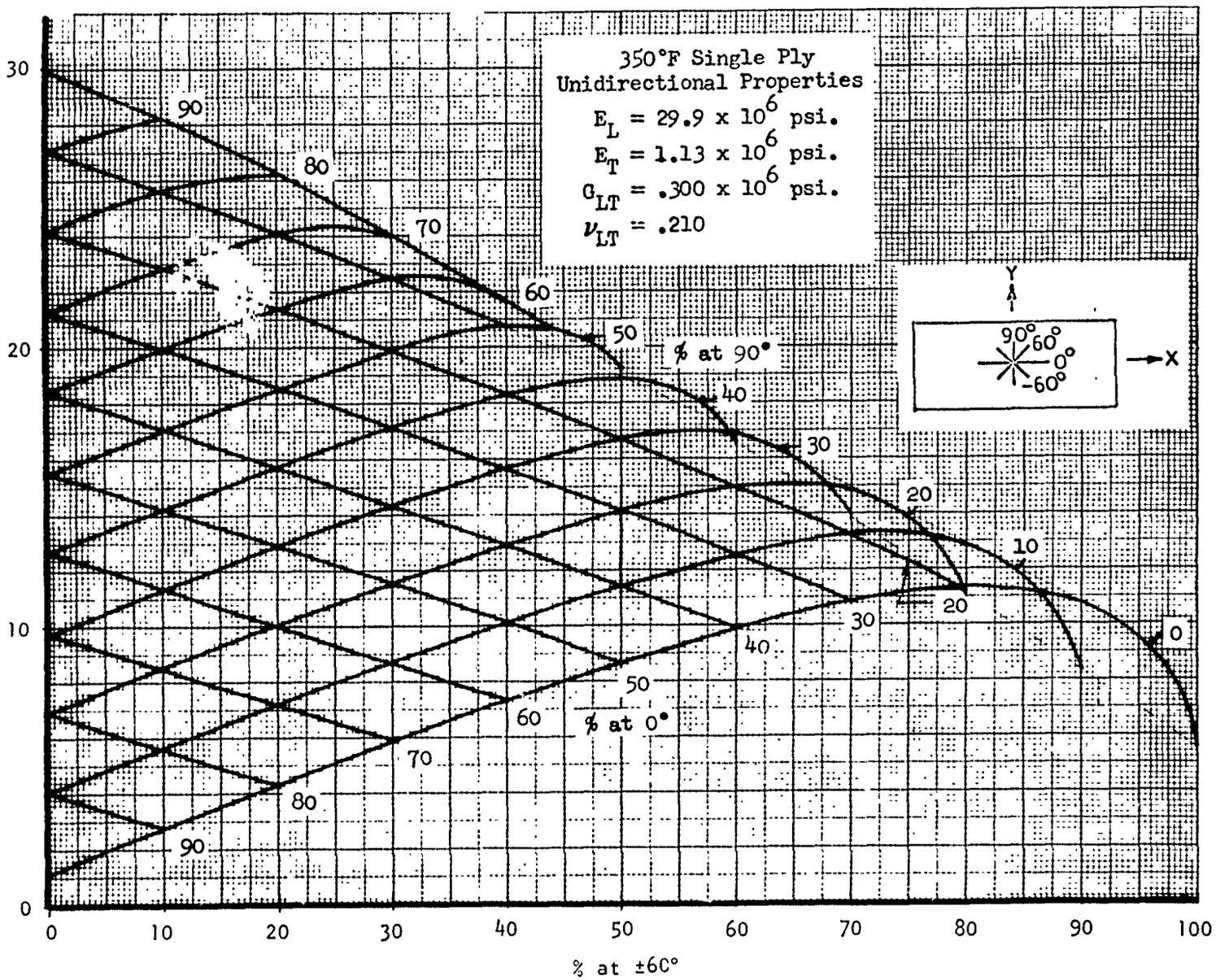


Figure 14. Laminate E_y vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F

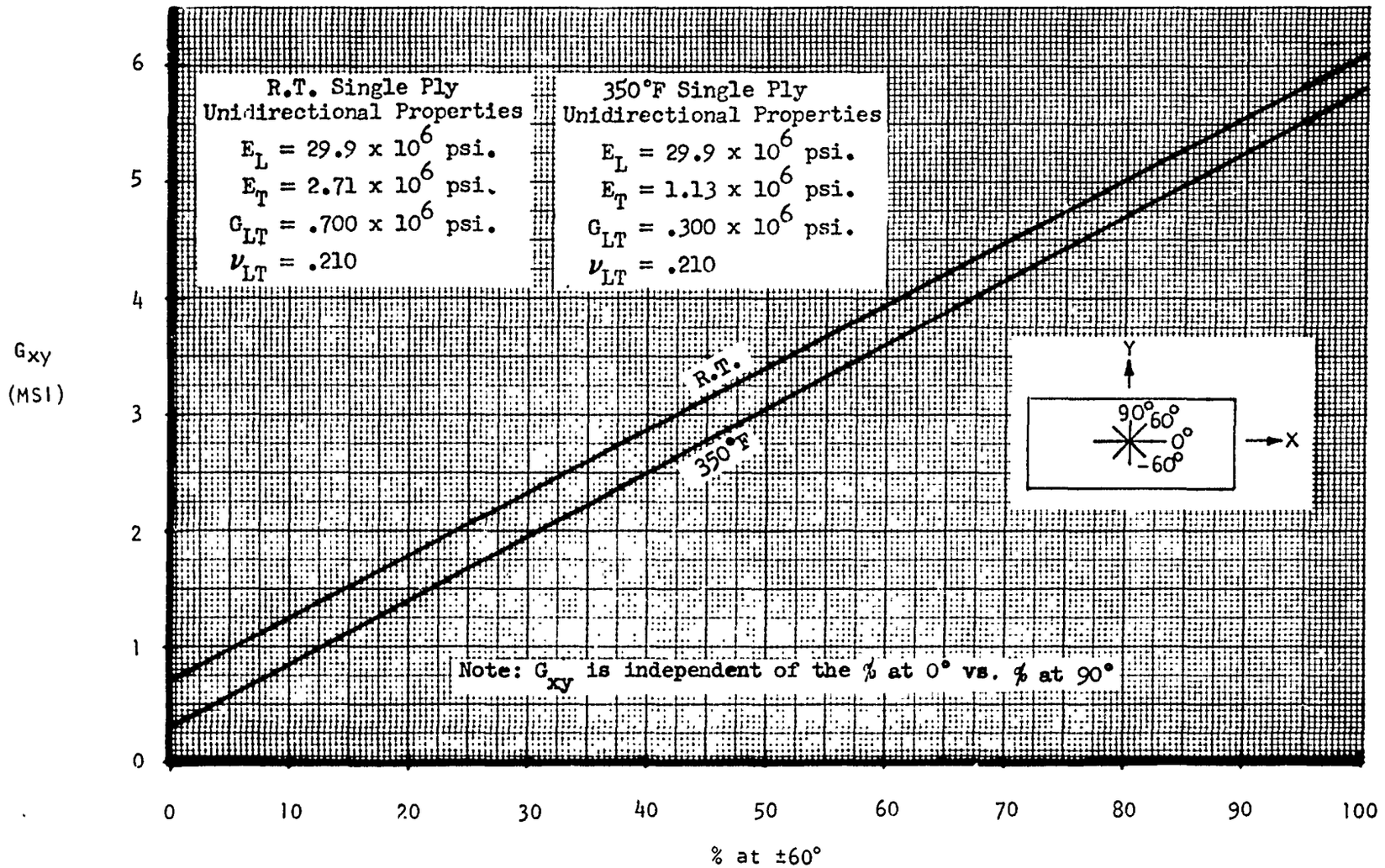


Figure 15. Laminate G_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT and 350° F

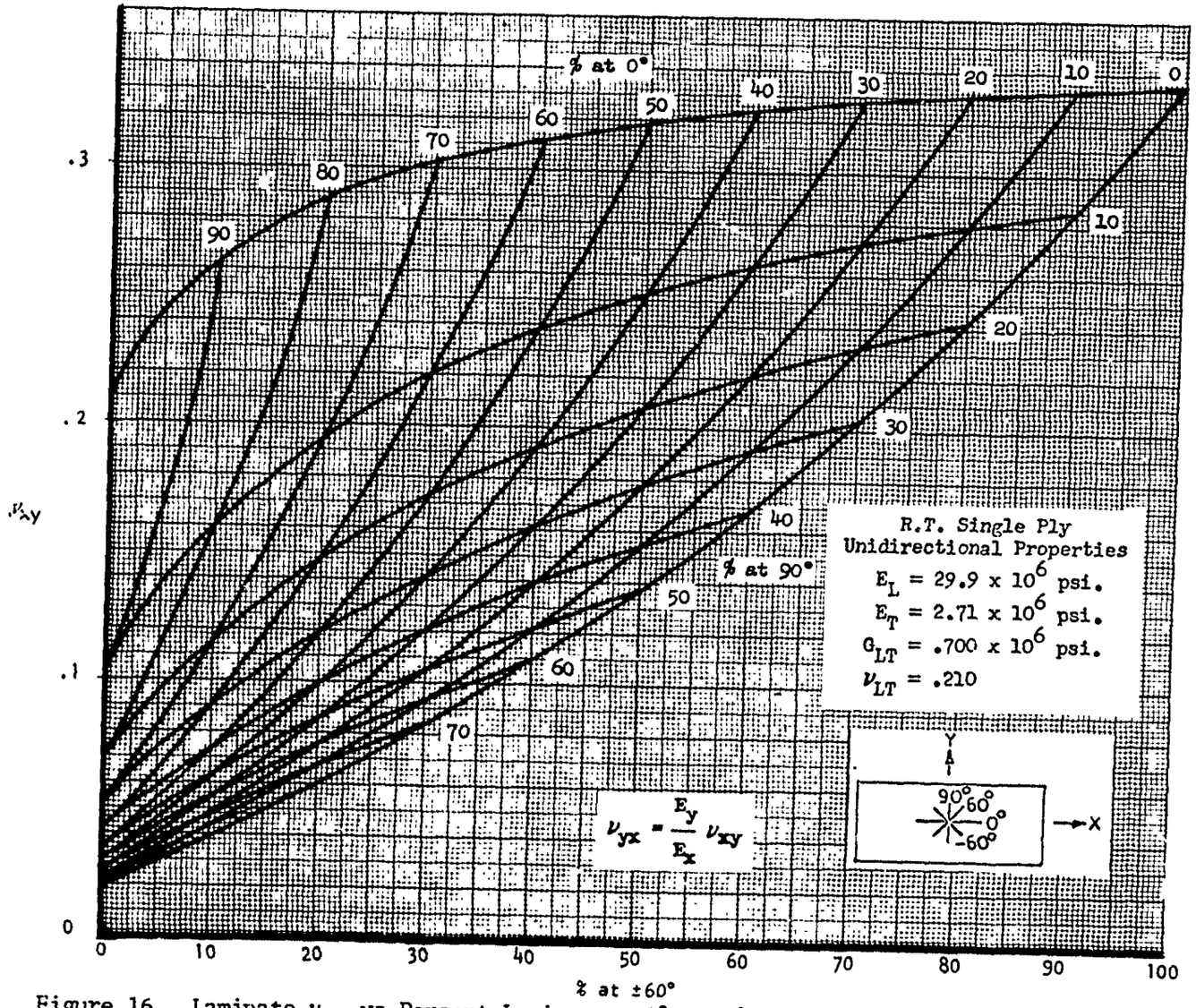


Figure 16. Laminate ν_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at RT

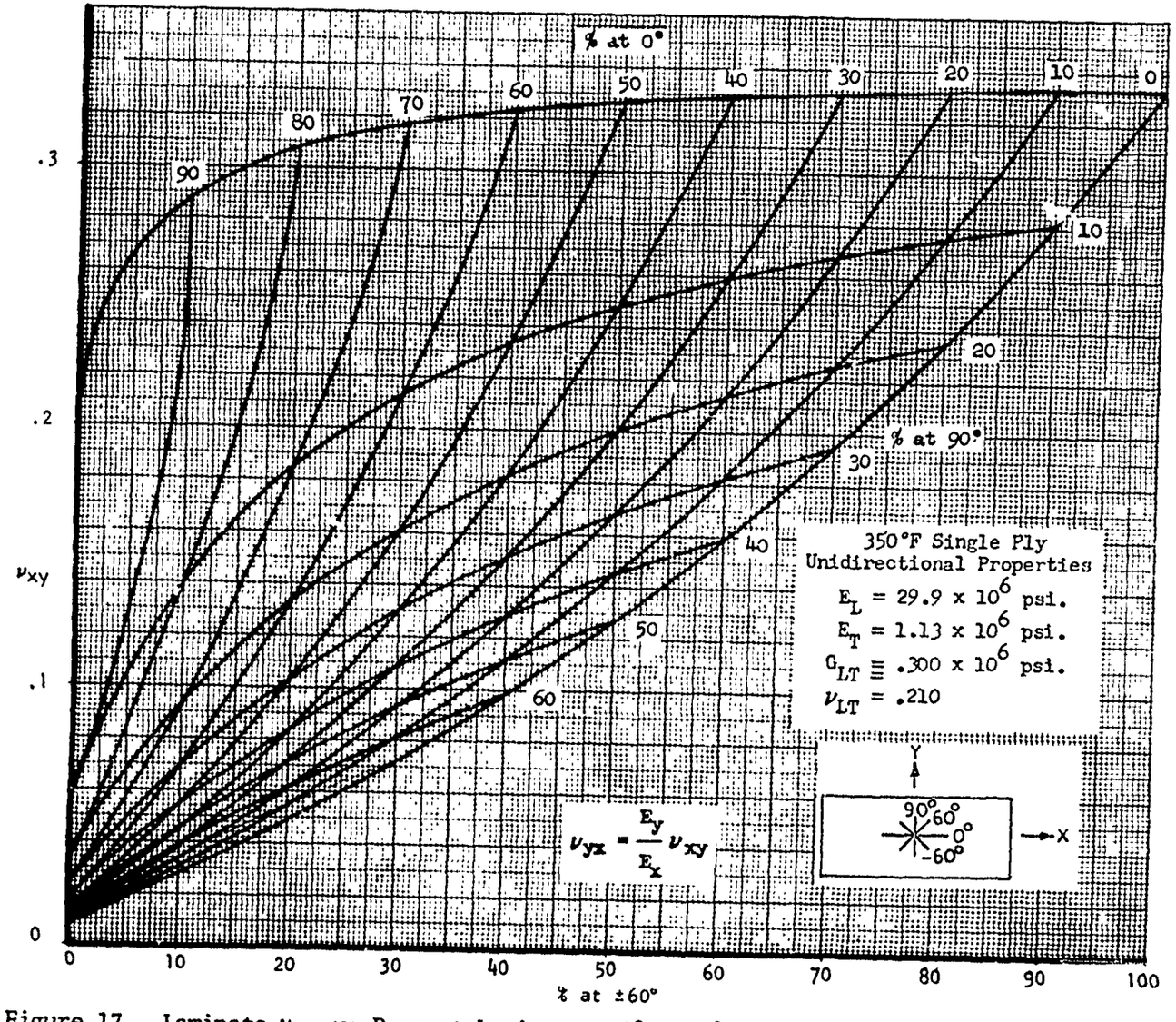


Figure 17. Laminate ν_{xy} vs Percent Laminae at 0°, ±60°, and 90° for Narmco 5505 at 350° F

SECTION III

FLAT PLATES

EQUATIONS OF MOTION

The equations of motion for the plate element shown in figure 18 are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \rho t \frac{\partial^2 u}{\partial t^2} \quad \left(\sum F_x = ma_x\right) \tag{25a}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = \rho t \frac{\partial^2 u}{\partial t^2} \quad \left(\sum F_y = ma_y\right) \tag{25b}$$

$$\frac{\partial M_y}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = 0 \quad \left(\sum M_y = 0\right) \tag{25c}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \quad \left(\sum M_x = 0\right) \tag{25d}$$

$$q + N_x \frac{\partial^2 w}{\partial x} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho t \frac{\partial^2 w}{\partial t^2}$$

$$\left(\sum F_z = ma_z\right) \tag{25e}$$

where the plate rotary inertia is considered to be negligible, ρ is the mass density of the plate, t is the plate thickness, the force and moment components are defined by equation 6, and the condition which each equation enforces for the element is indicated within the parentheses. Also, all plies are considered to have the same density, which is generally the case. The shear forces Q_x and Q_y can be eliminated from these expressions by combining equations 25c, 25d, and 25e to yield the relation

$$N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho t \frac{\partial^2 w}{\partial t^2} - q \tag{26}$$

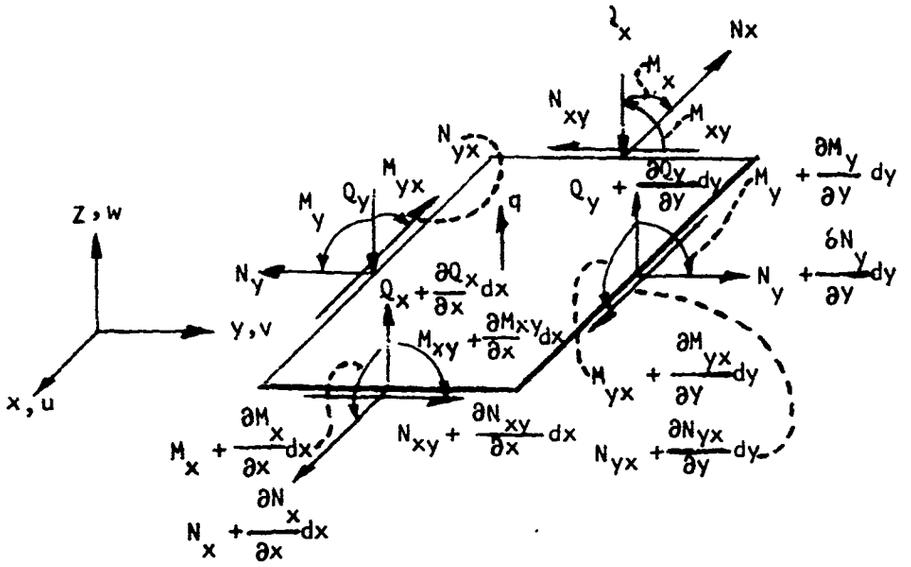


Figure 18. Plate Element Subjected to Force and Moment Components

If, as before, the discussion is limited to a laminate with a ply layup which is symmetric relative to its midsurface, then combining equations 7b and 25e and neglecting in-plane inertia terms yields

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{27a}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{27b}$$

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4\,D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2\,(D_{12}+2\,D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2}$$

$$+ 4\,D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = -\rho t\frac{\partial^2 w}{\partial t^2} + q \tag{27c}$$

$$+ N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2\,N_{xy}\frac{\partial^2 w}{\partial x \partial y}$$

This equation is based on the assumption that the normal deflections of the plate are small compared to its thickness, and the laminate ply orientation is symmetric relative to its midsurface. The latter assumption permits the uncoupling of the bending and membrane force and displacement components.

An examination of equation 27c indicates that except for cases when the laminate layup is composed totally of 0- and 90-degree plies, the D_{16} and D_{26} terms are nonzero. However, for thick laminates composed of many repeating sets of plies, the D_{16} and D_{26} stiffness terms tend toward zero.

This can be seen in figures 19, 20, 21, and 22, which plot the ratio of these stiffnesses to the D_{11} and D_{22} stiffnesses for 50-percent volume fraction boron/epoxy laminates. If we consider the average ply thickness to be 0.005 inch, we see that for the laminate orientations considered, the ratio $|D_{16}/D_{11}|$ or $|D_{26}/D_{11}|$ is less than 7 percent of D_{11} for thicknesses of 0.20 inch (40 plies) or greater. In addition, for thick laminates, the flexural stiffnesses can be defined in terms of the extensional constants for the basic ply set which is repeated to form the laminate. This is because each basic set of thick laminates undergoes nearly a uniform strain field during bending, as shown in figure 23, while this is not true for thin laminates.

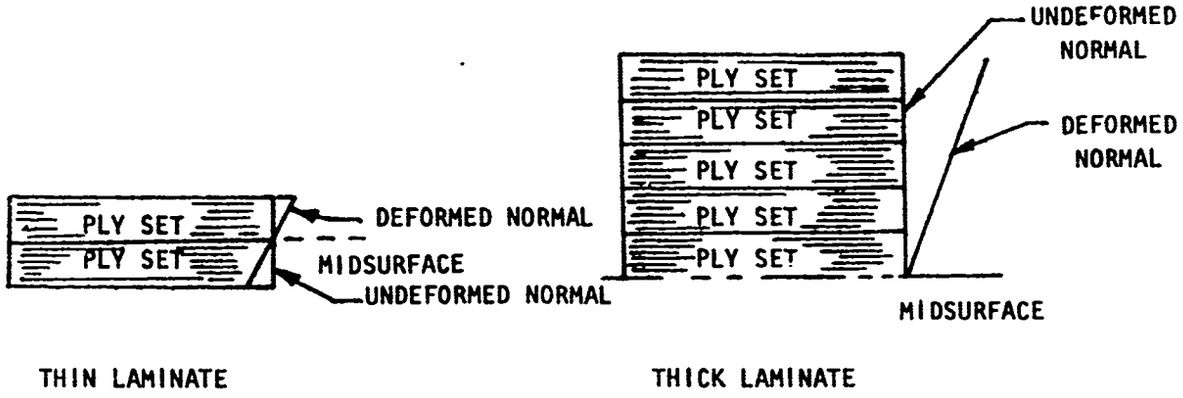


Figure 19. Thin and Thick Laminate Deformation

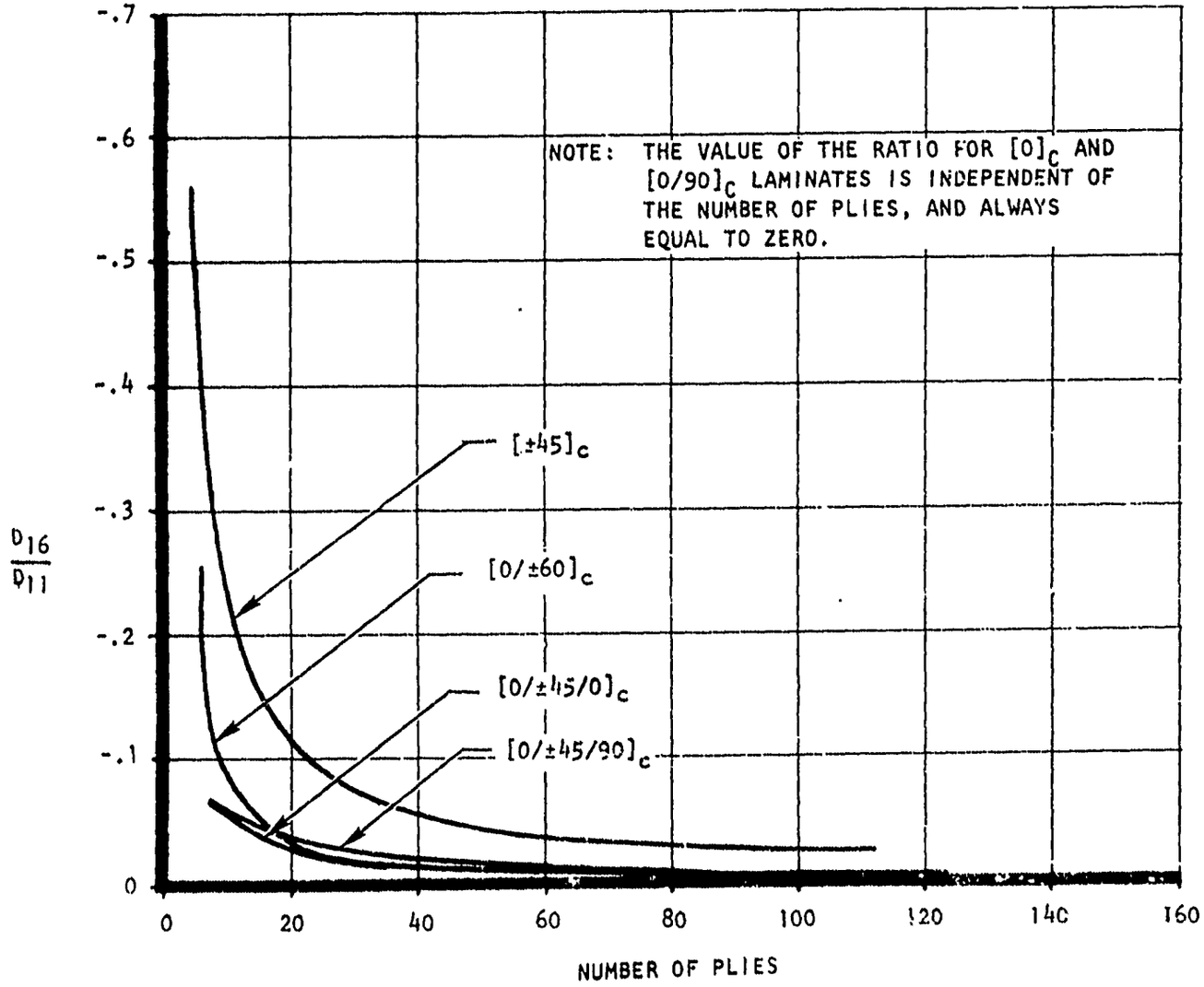


Figure 20. D_{16}/D_{11} vs Number of Plies for Several Laminate Orientations

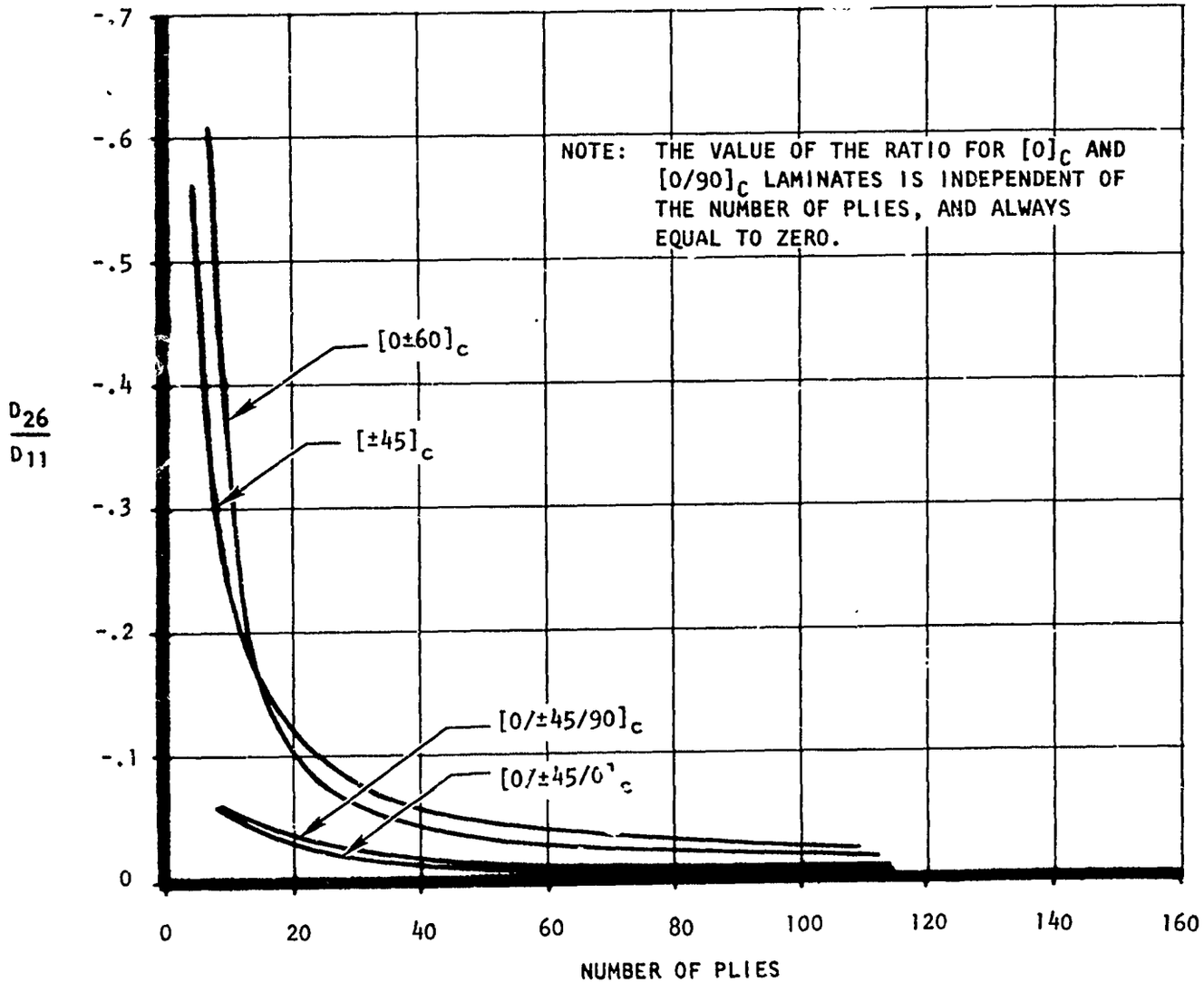


Figure 21. D_{26}/D_{11} vs Number of Plies for Several Laminate Orientations

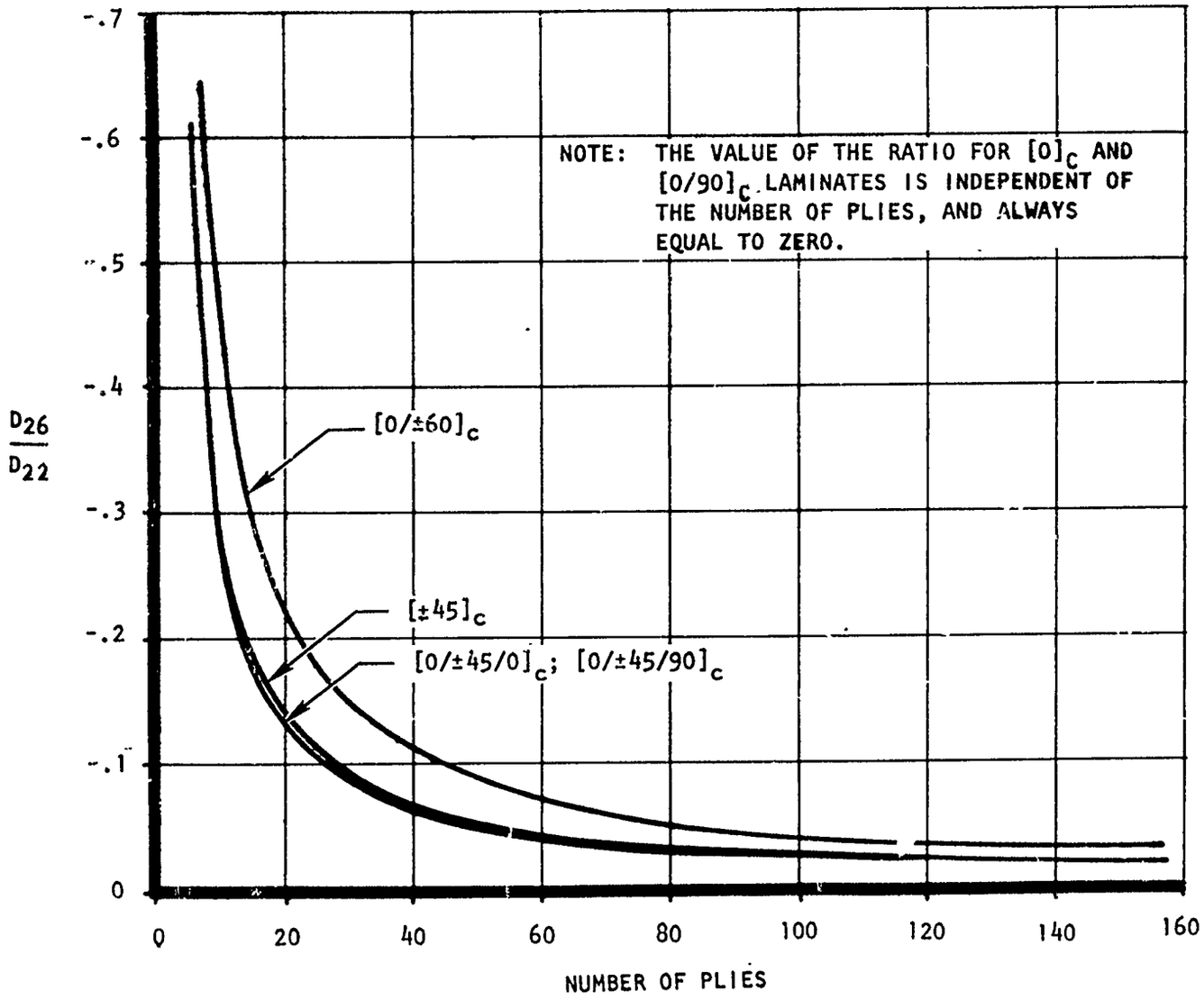


Figure 22. D_{26}/D_{22} vs Number of Plies for Several Laminate Orientations

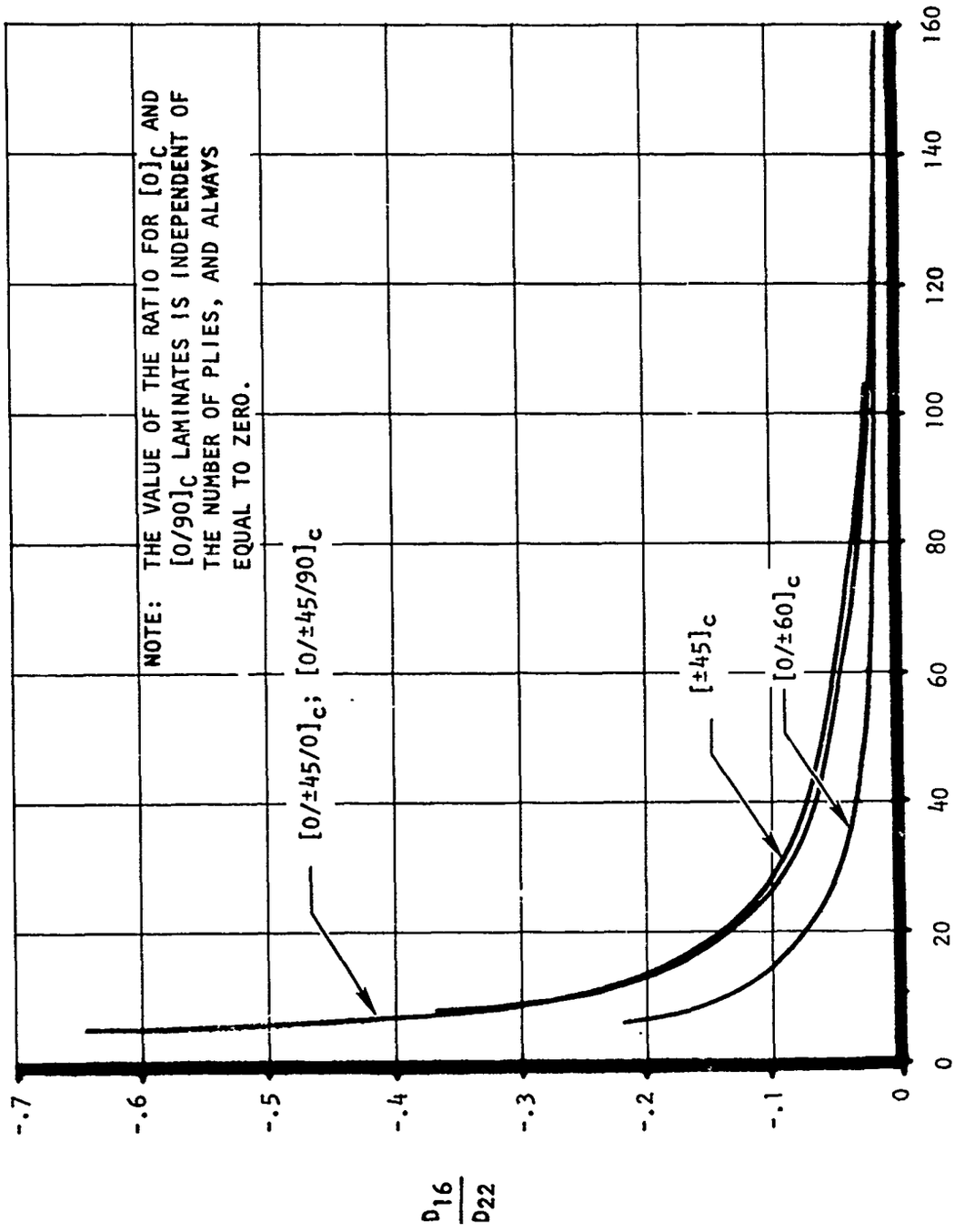


Figure 23. D_{16}/D_{22} vs Number of Plies for Several Laminate Orientations

Thus, for thick laminates and zero temperature change, the nonzero flexural stiffness can be defined by

$$D_{11} = \frac{E_x t^3}{12\,(1-\nu_{xy}\,\nu_{yx})}$$

$$D_{22} = \frac{E_y t^3}{12\,(1-\nu_{xy}\,\nu_{yx})} \tag{28}$$

$$D_{12} = D_{21} = \frac{\nu_{yx} E_x t^3}{12\,(1-\nu_{xy}\,\nu_{yx})}$$

$$D_{66} = \frac{G_{xy} t^3}{12}$$

where the elastic constants for each laminate are obtained from equation 13. Equation 27c reduces to the following classical governing equation for specially orthotropic plates

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2 D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = -\rho t \frac{\partial^2 w}{\partial t^2} + q$$

$$+ N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} \tag{29}$$

where

$$D_1 = D_{11}$$

$$D_3 = D_{12} + 2 D_{66}$$

$$D_2 = D_{22}$$

A broad class of closed-form orthotropic plate solutions to equation 29 are available in the literature, and are valid for the analysis of thick, laminated, filamentary composite structures.

PRESSURE PANELS

The classical theory (reference 11) for the deflection and moments of a uniformly loaded, simply supported, specially orthotropic laminated plate can be obtained from equation 29 by setting

$$N_x = N_y = N_{xy} = \frac{\partial^2 w}{\partial t^2} = 0$$

and the plate's normal deflection W in the form

$$w = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{30}$$

and the uniform normal pressure by

$$q = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{16\, q}{\pi^2\, mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{31}$$

then substituting equations 30 and 31 into equation 29 and solving for A_{mn}. This procedure results in the following relationship for the plate's normal deflection:

$$w = 16 \frac{q\, b^4}{\pi^6} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{32}$$

where

$$A_{mn} = \frac{16\, q\, b^4}{\pi^6 mn \left[D_{11} \left(\frac{mb}{a}\right)^4 + 2(D_{12}+2D_{66}) \left(\frac{mnb}{a}\right)^2 + D_{22}\, n^4 \right]}$$

The stresses of any i^{th} ply at a distance z from the midsurface of the laminate, which correspond to the deflection field described by equation 32, can be obtained from equation 5 as

$$\sigma_x^{(i)} = \frac{-E_y^{(i)}\, z}{1 - \nu_{xy}^{(i)}\, \nu_{yx}^{(i)}} \sum_{m=1,3,5} \sum_{n=1,3,5} A_{mn} \pi^2 \left\{ \frac{m^2}{a^2} + \nu_{yx}^{(k)} \frac{n^2}{b^2} \right\} \cdot \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{33}$$

$$\sigma_y^{(i)} = \frac{-E_y^{(i)} z}{1 - \nu_{xy}^{(i)} \nu_{yx}^{(i)}} \cdot \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \pi^2 \left\{ \frac{n^2}{b^2} + \nu_{xy}^{(k)} \frac{m^2}{a^2} \right\} \cdot \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{34}$$

for 0- and 90-degree plies. For plies of other orientations, the contributions of $\bar{Q}_{26}$ and $\bar{Q}_{16}$ must be considered.

For the case when in-plane compression forces N_x and N_y are also present, A_{mn} becomes

$$A_{mn} = \frac{16\, q\, b^4}{\pi^6 mn \left\{ \left[D_{11} \left(\frac{mb}{a}\right)^4 + 2(D_{12}+2D_{66}) \left(\frac{mnb}{a}\right)^2 + D_{22} n^4 \right] - \left(\frac{b}{\pi}\right)^2 \left[N_x \left(\frac{mb}{a}\right)^2 + N_y n^2 \right] \right\}} \tag{35}$$

UNIAXIAL AND BIAXIAL COMPRESSION

The classical solution for the buckling of specially orthotropic, simply supported plates is obtained from equation 29 by setting

$$N_{xy} = q = \frac{\partial^2 w}{\partial t^2} = 0$$

and taking w in the form

$$w = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{36}$$

where m and n denote the number of half-wave lengths along the X and Y axes, respectively.

Noting that the forces N_x and N_y are compression, we obtain from equation 29

$$(N_x)_{cr} = \frac{k_x \pi^2 D_{11}}{b^2} \tag{37}$$

where

$$k_x = \left\{ \frac{\left[m^4 \left(\frac{b}{a}\right)^4 + 2\left(\frac{D_{12}}{D_{11}} + 2\,\frac{D_{66}}{D_{11}}\right) m^2 n^2 \left(\frac{b}{a}\right)^2 + \frac{D_{22}}{D_{11}} n^4 \right]}{\left(\frac{mb}{a}\right)^2 + \phi n^2} \right\}$$

$$\phi = \frac{N_y}{N_x}$$

and m and n are the half-wave length integers which minimize k_x. For the case when only N_x is present, i.e., $\phi = 0$, and the plate is very long $(a/b \gg 1)$, we have, after noting that k_x is minimized in this case for n = 1,

$$\frac{dk_x}{dm} = 2m\left(\frac{b}{a}\right)^2 - 2\,\frac{\left(\frac{D_{22}}{D_{11}}\right)}{m^3\left(\frac{b}{a}\right)^2} = 0$$

or $(k_x)_{min}$ occurs when

$$m = \left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\frac{a}{b}\right)$$

when m is a whole number and we see for a long plate that k_x becomes

$$k_x = 2\left\{1 + \frac{(D_{12} + 2\,D_{66})}{\sqrt{D_{11}\,D_{22}}}\right\} \tag{38}$$

Figures 24 and 25 are plots of $N_{x_{cr}}$ for $[0/\pm45/0]_{2S}$ laminates at room temperature and 350° F as a function of panel aspect ratio, panel width, and ratio of N_y to N_x. These curves were obtained from equation 37.

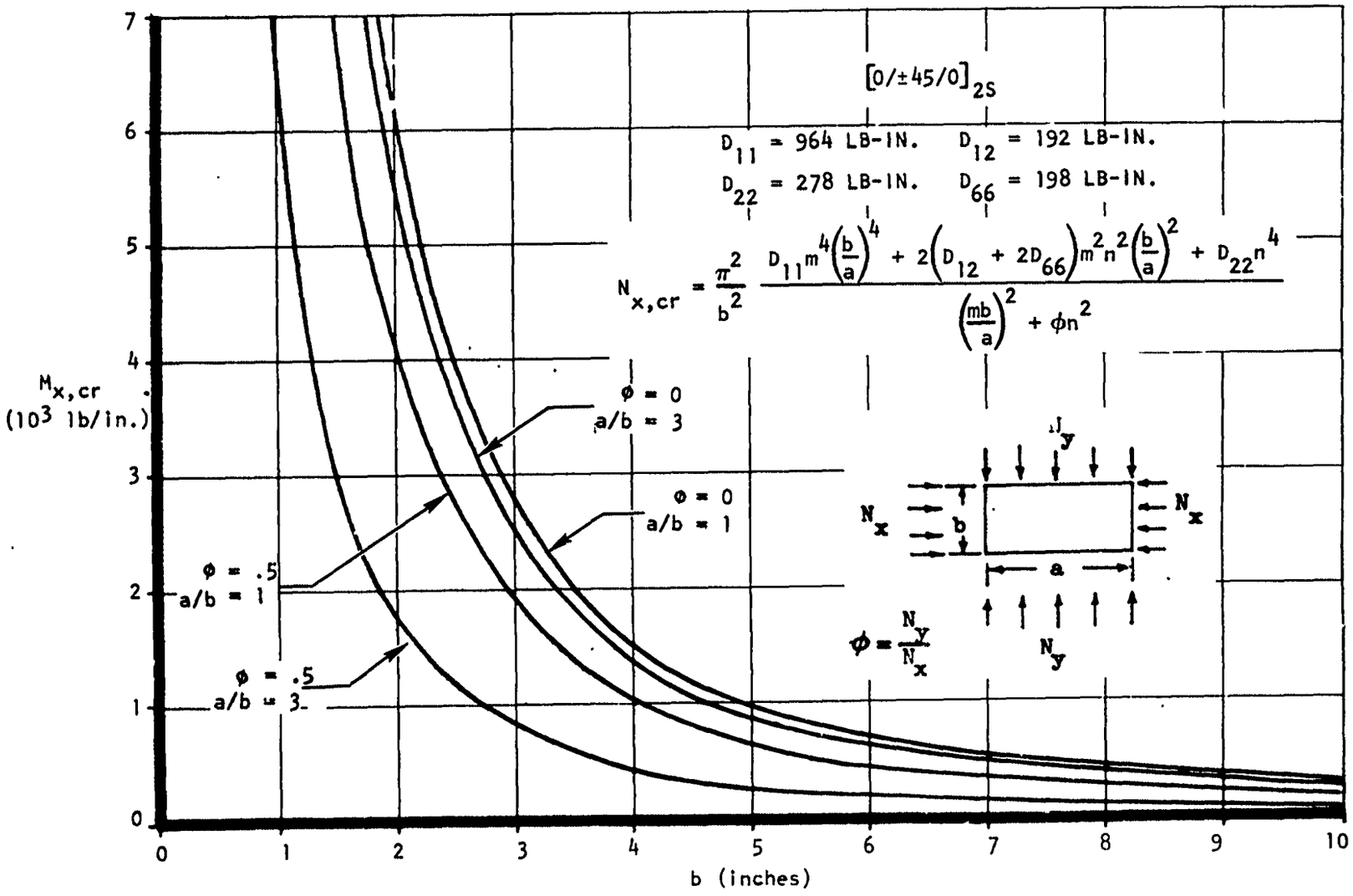


Figure 24. Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (RT)

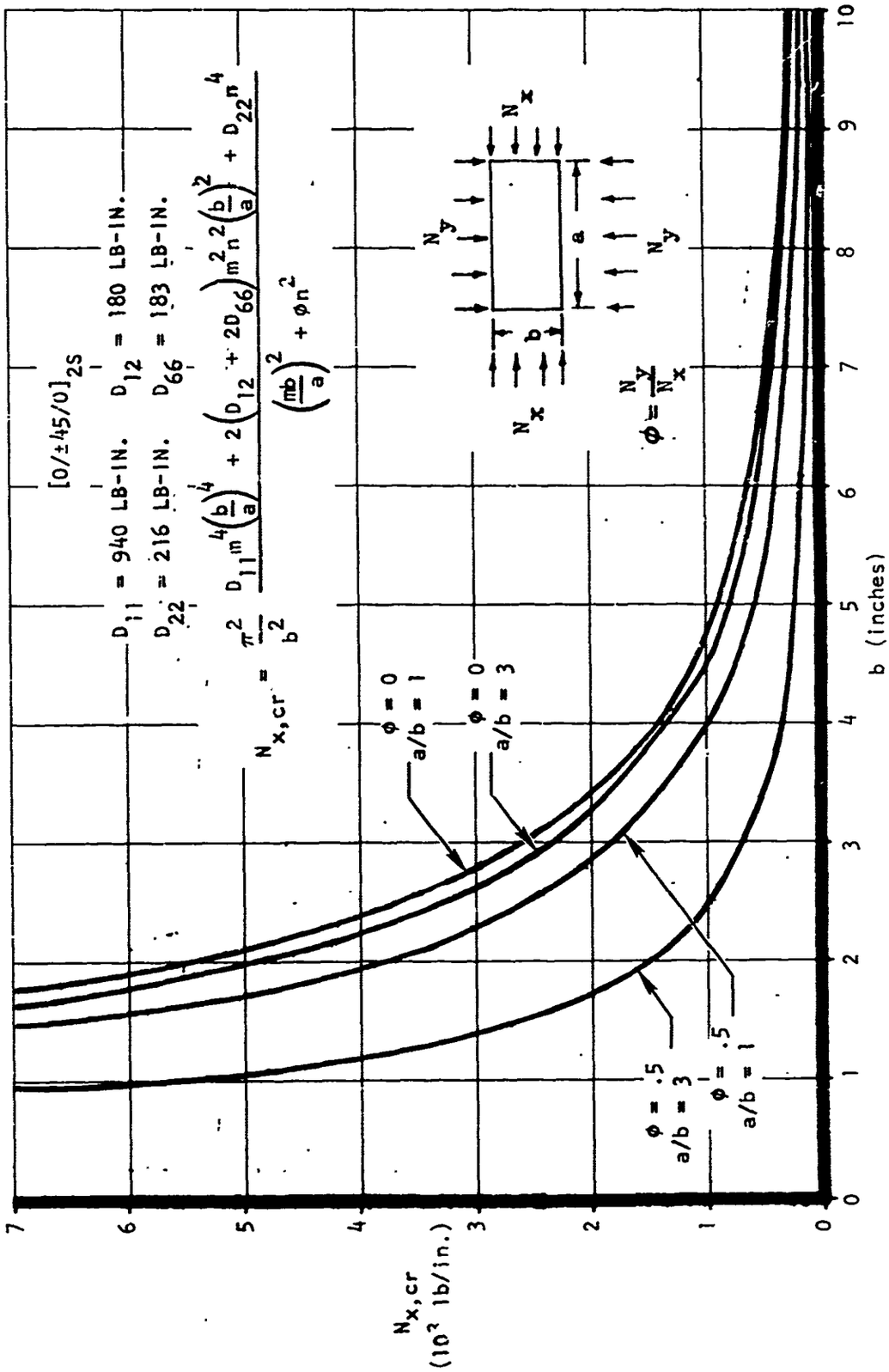


Figure 25. Buckling Load $N_{x,cr}$ vs Width b for a Simply Supported Plate With N_y (350° F)

NATURAL FREQUENCIES

If we seek the fundamental frequencies associated with various m and n mode shapes for a simply supported plate, w is taken in the form

$$w(x, y, t) = e^{i\omega_{mn}t} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{39}$$

where ω_{mn} is the angular velocity in radians per second. Substitution of equation 39 into equation 29 and setting the in-plane forces and normal pressure q to zero yields

$$\omega_{mn} = \left\{ \frac{D_1 \left(\frac{m\pi}{a}\right)^4 + 2 D_3 \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + D_2 \left(\frac{n\pi}{b}\right)^4}{\rho t} \right\}^{\frac{1}{2}} \tag{40}$$

SECTION IV

ANALYSIS OF ORTHOTROPIC SANDWICH PLATES AND SHELL STRUCTURES

CONSTITUTIVE RELATIONS

In solid plates and shells, shear strain is generally neglected. But in sandwich plates and shells, shear strain in core needs to be considered. The sandwich element is composed of two face plates and a core as shown in figure 26. The distance between the centroids of the two faces is

$$h = c + t_f \tag{41}$$

Consider a cylindrical sandwich shell element as shown in figure 27. Let X, Y, Z be a set of orthogonal axes with X along the genatrix of the cylinder, Y tangent to the cylindrical surface, and Z normal to the cylindrical surface, and u, v, w be displacements along these three axes, respectively. Let N_x, N_y, N_{xy}, denote the sectional forces; M_x, M_y, M_{xy} the sectional moments; and Q_x, Q_y, the transverse shear forces per unit length of the element. This element is subject to a distributed lateral load q per unit area.

The equations of motion of this element are written as

$$\frac{\partial M_y}{\partial x} + \frac{\partial M_{yx}}{\partial y} - Q_x = I \frac{\partial^2 \beta_x}{\partial t^2} \tag{42}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I \frac{\partial^2 \beta_y}{\partial t^2} \tag{43}$$

$$q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - \frac{N_y}{R} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = M \frac{\partial^2 w}{\partial t^2} \tag{44}$$

where I denotes the rotary moment of inertia, M the mass per unit area of the element, R the radius, t the time, β_x and β_y rotations as shown in figure 28.

Let ρ_f and ρ_c' denote the mass densities of the face and the core, respectively.

$$I \cong \rho_f t_f \frac{\left(c^2 + ct_f\right)}{2} + \rho_c' \frac{c^3}{12} \tag{45}$$

$$M = 2t_f \rho_f + c\rho_c' \tag{46}$$

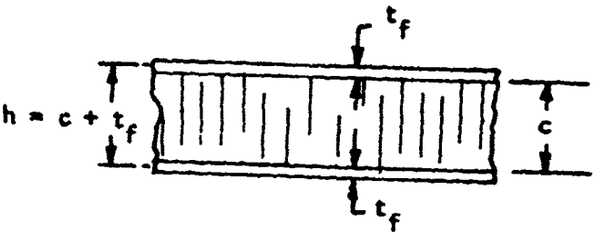


Figure 26. Section of a Sandwich Plate

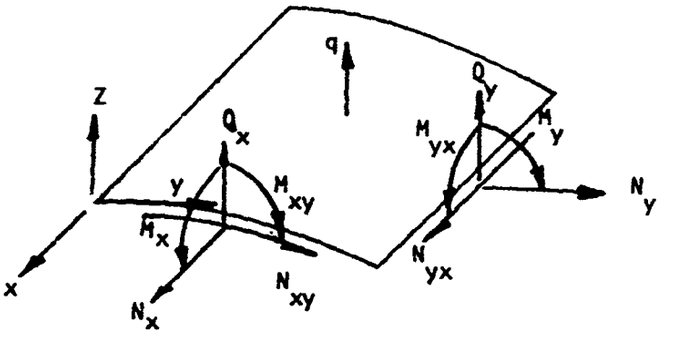


Figure 27. Positive Direction of Sectional Forces and Moments

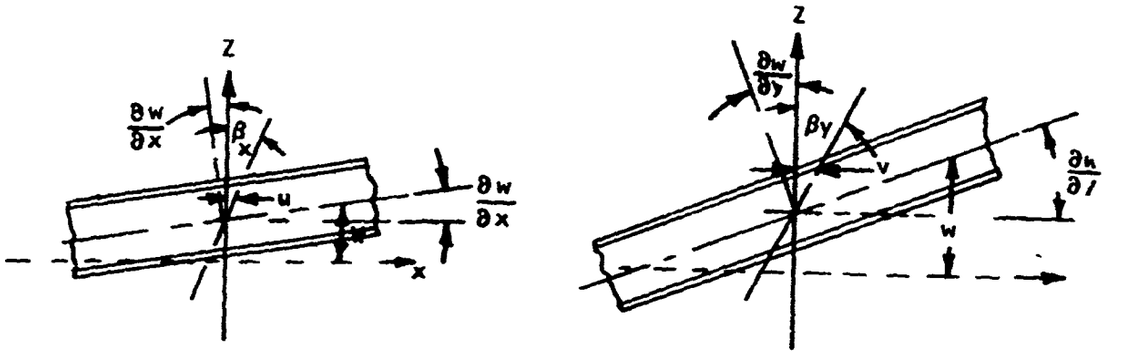


Figure 28. Rotations Due to Shear

NATURAL FREQUENCIES

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where ω_{mn} is the angular velocity in radians per second. Substitution of equation 39 into equation 29 and setting the in-plane forces and normal pressure q to zero yields

$$\omega_{mn} = \left\{ \frac{D_1 \left(\frac{m\pi}{a}\right)^4 + 2 D_3 \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + D_2 \left(\frac{n\pi}{b}\right)^4}{\rho t} \right\}^{\frac{1}{2}} \tag{40}$$

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$$h = c + t_f \tag{41}$$

Consider a cylindrical sandwich shell element as shown in figure 27. Let X, Y, Z be a set of orthogonal axes with X along the genatrix of the cylinder, Y tangent to the cylindrical surface, and Z normal to the cylindrical surface, and u, v, w be displacements along these three axes, respectively. Let N_x, N_y, N_{xy}, denote the sectional forces; M_x, M_y, M_{xy} the sectional moments; and Q_x, Q_y, the transverse shear forces per unit length of the element. This element is subject to a distributed lateral load q per unit area.

The equations of motion of this element are written as

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$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I \frac{\partial^2 \beta_y}{\partial t^2} \tag{43}$$

$$q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} - \frac{N_y}{R} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = M \frac{\partial^2 w}{\partial t^2} \tag{44}$$

where I denotes the rotary moment of inertia, M the mass per unit area of the element, R the radius, t the time, β_x and β_y rotations as shown in figure 28.

Let ρ_f and ρ_c' denote the mass densities of the face and the core, respectively.

$$I \cong \rho_f t_f \frac{\left(c^2 + c t_f\right)}{2} + \rho_c' \frac{c^3}{12} \tag{45}$$

$$M = 2 t_f \rho_f + c \rho_c' \tag{46}$$

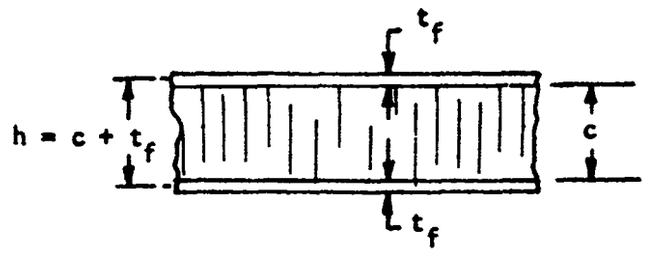


Figure 26. Section of a Sandwich Plate

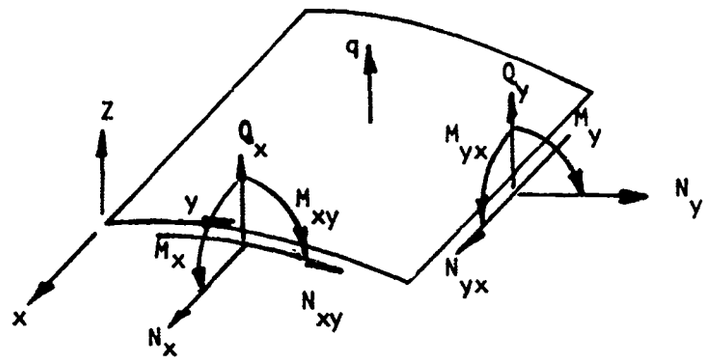


Figure 27. Positive Direction of Sectional Forces and Moments

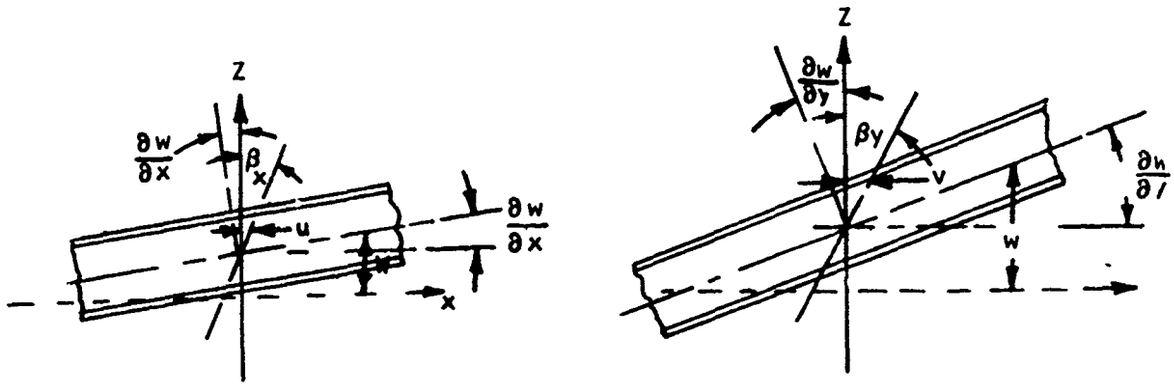


Figure 28. Rotations Due to Shear

Referring to figure 28, we see that the u and v components can be taken in the form

$$u(z) = \beta_x z; \quad v(z) = \beta_y z \tag{47}$$

Assume the bending moment to be taken solely by the faces and shear solely by the core, then

$$\begin{aligned} \tau_{yz} &= G'_{cy}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = G'_{cy}\left(\beta_y+\frac{\partial w}{\partial y}\right) \\ \tau_{xz} &= G'_{cx}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) = G'_{cx}\left(\beta_x+\frac{\partial w}{\partial x}\right) \end{aligned} \tag{48}$$

$$\begin{aligned} Q_y &\cong \int_{-\frac{c}{2}}^{\frac{c}{2}} \tau_{yz}\,dz = G'_{cy}\left(\beta_y+\frac{\partial w}{\partial y}\right) \\ Q_x &\cong \int_{-\frac{c}{2}}^{\frac{c}{2}} \tau_{xz}\,dz = G'_{cx}\left(\beta_x+\frac{\partial w}{\partial x}\right) \end{aligned} \tag{49}$$

The face sheets are considered to be specially orthotropic and thus their stress-strain relation is given by

$$\sigma_{xx} = \frac{E_x}{1-\nu_{xy}\nu_{yx}}(e_x+\nu_{yx}e_y) = \frac{E_x}{1-\nu_{xy}\nu_{yx}}\left(\frac{\partial u}{\partial x}+\nu_{yx}\frac{\partial v}{\partial y}\right)$$

$$\cong \frac{E_x}{1-\nu_{xy}\nu_{yx}}\frac{c}{2}\left(\frac{\partial \beta_x}{\partial x}+\nu_{yx}\frac{\partial \beta_y}{\partial y}\right) \tag{50}$$

Similarly,

$$\sigma_{yy} \cong \frac{E_y}{1-\nu_{xy}\nu_{yx}}\frac{c}{2}\left(\frac{\partial \beta_y}{\partial y}+\nu_{xy}\frac{\partial \beta_x}{\partial x}\right) \tag{51}$$

$$\tau_{xy} \cong G_{xy}\frac{c}{2}\left(\frac{\partial \beta_x}{\partial y}+\frac{\partial \beta_y}{\partial x}\right) \tag{52}$$

The section moments are obtained by integration.

$$M_x = \int_{\frac{c}{2}}^{\frac{c}{2}+t_f} \sigma_{xx} z\,dz + \int_{-(\frac{c}{2}+t_f)}^{-\frac{c}{2}} \sigma_{xx} z\,dz = D_{11}\left(\frac{\partial \beta_x}{\partial x}+\nu_{yx}\frac{\partial \beta_y}{\partial y}\right) \tag{53}$$

where

$$D_{11} = \frac{E_x c^2 t_f}{2\left(1-\nu_{xy}\nu_{yx}\right)}\left(1+\frac{t_f}{c}\right)$$

Similarly,

$$M_y = D_{22}\left(\frac{\partial \beta_y}{\partial y} + \nu_{xy}\frac{\partial \beta_x}{\partial x}\right) \tag{54}$$

where

$$D_{22} = D_{11}\frac{E_y}{E_x}$$

$$M_{xy} = D_{66}\left(\frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x}\right) \tag{55}$$

where

$$D_{66} = t_f \frac{G_{xy}}{2}\left(c^2 + t_f c\right)$$

Substituting equations 53, 54, and 55 into equations 42 and 43 we obtain

$$D_{11}\left(\frac{\partial^2 \beta_x}{\partial x^2} + \nu_{yx}\frac{\partial^2 \beta_y}{\partial x \partial y}\right) + D_{66}\left(\frac{\partial^2 \beta_y}{\partial y^2} + \frac{\partial^2 \beta_y}{\partial x \partial y}\right) - Q_x = I\frac{\partial^2 \beta_x}{\partial t^2} \tag{56}$$

$$D_{22}\left(\frac{\partial^2 \beta_y}{\partial y^2} + \nu_{xy}\frac{\partial^2 \beta_x}{\partial x \partial y}\right) + D_{66}\left(\frac{\partial^2 \beta_x}{\partial x \partial y} + \frac{\partial^2 \beta_y}{\partial x^2}\right) - Q_y = I\frac{\partial^2 \beta_y}{\partial t^2} \tag{57}$$

Now let us consider the case of a simply supported rectangular plate. Let

$$w = e^{i\omega t} A_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{58}$$

$$Q_x = e^{i\omega t} B_{mn} \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{59}$$

$$Q_y = e^{i\omega t} C_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \tag{60}$$

$$q = e^{i\omega t} q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{61}$$

From equations 49, 58, 59, and 60, we find

$$\beta_y = e^{i\omega t}\left(\frac{C_{mn}}{cG'_{cy}} - \frac{n\pi}{b} A_{mn}\right) \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{62}$$

$$\beta_x = e^{i\omega t}\left(\frac{B_{mn}}{cG'_{cx}} - \frac{m\pi}{a} A_{mn}\right) \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{63}$$

Substituting equations 59, 62, and 63 into equation 56 and simplifying yields

$$B_{mn}\left\{1 + \frac{D_{11}}{cG'_{cx}} \bar{m}^2 + \frac{D_{66}}{cG'_{cx}} \bar{n}^2 - I\frac{\omega^2}{cG'_{cx}}\right\} + C_{mn}\left\{\frac{\nu_{yx}}{cG'_{cy}} D_{11}\bar{m}\bar{n} + \frac{D_{66}}{cG'_{cy}} \bar{m}\bar{n}\right\} - A_{mn} D_{11}\left\{\bar{m}^3 + \nu_{yx} D_{11} mn^2 + 2D_{66}\bar{m}\bar{n}^2 - I\omega^2\bar{m}\right\} = 0 \tag{64}$$

where

$$\bar{m} = \frac{m\pi}{a} \text{ and } \bar{n} = \frac{n\pi}{b} \tag{65}$$

Equation 64 can be written in the following compacted form:

$$a_{11} A_{mn} + a_{12} B_{mn} + a_{13} C_{mn} = 0 \tag{66}$$

where a_{11}, a_{12}, and a_{13} are the coefficient of A_{mn}, B_{mn}, and C_{mn}, respectively.

Similarly, substituting equations 59, 62, and 63 into equation 57, we obtain

$$a_{21} A_{mn} + a_{22} B_{mn} + a_{23} C_{mn} = 0 \tag{67}$$

where

$$a_{21} = D_{22}\,\bar{n}^3 + \left(2D_{66} + \nu_{xy}\,D_{22}\right)\bar{n}\bar{m}^2 - \bar{n}\,I\omega^2$$

$$a_{22} = \frac{\bar{m}\bar{n}}{cG'_{cy}}\left(D_{22}\,\nu_{xy} + D_{66}\right) \tag{68}$$

$$a_{23} = -\frac{1}{cG'_{cy}}\left(cG'_{cy} + D_{22}\,\bar{n}^2 + D_{66}\,\bar{m}^2 - I\omega^2\right)$$

A plate can be considered as a circular shell with infinite radius. R in equation 44 becomes infinity. Let $N_{xy} = 0$. Then eliminating Q_x and Q_y in equation 44 by equations 56 and 57, we obtain

$$q + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} - I\frac{\partial^3 \beta_x}{\partial x \partial t^2} + D_{11}\left(\frac{\partial^3 \beta_x}{\partial x^3} + \nu_{yx}\frac{\partial^3 \beta_y}{\partial x^2 \partial y}\right)$$

$$+ 2\,D_{66}\left(\frac{\partial^3 \beta_x}{\partial x \partial y^2} + \frac{\partial^3 \beta_y}{\partial x^2 \partial y}\right) + D_{22}\left(\frac{\partial^3 \beta_y}{\partial x^3} + \nu_{xy}\frac{\partial^3 \beta_y}{\partial x \partial y^2}\right) - I\frac{\partial^3 \beta_y}{\partial t^2 \partial y} = M\frac{\partial^2 w}{\partial t^2} \tag{69}$$

Substituting equations 58, 62, and 63 into the preceding equation yields

$$q_{mn} - \left(a_{31} + N_x\bar{m}^2 + N_y\bar{n}^2\right)A_{mn} + a_{32}\,B_{mn} + a_{33}\,C_{mn} = 0 \tag{70}$$

where

$$a_{31} = -\bar{m}^2\,I\omega^2 + D_{11}\,\bar{m}^4 + \bar{m}^2\bar{n}^2\left(\nu_{yx}D_{11} + 4D_{66} + \nu_{xy}D_{22}\right)$$

$$+ D_{22}\,\bar{n}^4 - I\omega^2\bar{n}^2 - \omega^2 M$$

$$a_{32} = \frac{\bar{m}}{cG'_{cx}}\left[-I\omega^2 + D_{11}\,\bar{m}^2 + \bar{n}^2\left(2D_{66} + \nu_{xy}D_{22}\right)\right] \tag{71}$$

$$a_{33} = \frac{\bar{n}}{cG'_{cy}}\left[-I\omega^2 + D_{22}\,\bar{n}^2 + \bar{m}^2\left(2D_{66} + \nu_{yx}D_{11}\right)\right]$$

From equations 66, 67, and 70, we obtain

$$C_{mn} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{12}a_{23} - a_{22}a_{13}} A_{mn} \tag{72}$$

$$B_{mn} = \frac{a_{13}a_{21} - a_{11}a_{23}}{a_{12}a_{23} - a_{22}a_{13}} A_{mn} \tag{73}$$

$$A_{mn} = \frac{q_{mn}\left(a_{22}a_{13}-a_{12}a_{23}\right)}{\left(a_{31}+\bar{m}^2N_x+\bar{n}^2N_y\right)\left(a_{22}a_{13}-a_{12}a_{23}\right) + a_{32}\left(a_{13}a_{21}-a_{11}a_{23}\right) + a_{33}\left(a_{11}a_{22}-a_{12}a_{21}\right)} \tag{74}$$

Let us define the following nondimensional ratios:

$$\alpha = \sqrt{\frac{E_x}{E_y}}, \quad r = \frac{G'_{cx}}{G'_{cy}}$$

$$\beta = \alpha\nu_{yx} + 2\gamma, \quad \gamma = \frac{D_{66}}{\sqrt{D_{11}D_{22}}} \tag{75}$$

$$V = \left(\frac{\pi}{b}\right)^2 \frac{\sqrt{D_{11}D_{22}}}{cG'_{cx}}$$

$$k_x = \frac{N_x\left(\frac{b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}, \quad k_y = \frac{N_y\left(\frac{b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}$$

Then the coefficients of equations 66, 67, and 70 may be written as

$$a_{11} = -\left(\frac{\pi}{b}\right)^3 \sqrt{D_{11}D_{22}} \left[\left(\frac{mb}{a}\right)^3 \alpha + mn^2\left(\frac{b}{a}\right)\beta - \frac{\mathrm{Im}\left(\frac{\omega b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}\left(\frac{b}{a}\right)\right]$$

$$a_{12} = 1 + m^2\alpha V\left(\frac{b}{a}\right)^2 + n^2\gamma V - \frac{I\omega^2}{cG'_{cx}}$$

$$a_{13} = mn\left(\frac{b}{a}\right)V(\beta-\gamma)\,r$$

$$a_{21} = \left(\frac{\pi}{b}\right)^3\sqrt{D_{11}D_{22}}\left[\frac{n^3}{\alpha}+nm^2\beta\left(\frac{b}{a}\right)^2-\frac{I\rho\left(\frac{\omega b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}\right]$$

$$a_{22} = -mn\left(\frac{b}{a}\right)V(\beta-\gamma)$$

$$a_{23} = -1-rV\left[\frac{n^2}{\alpha}+\gamma m^2\left(\frac{b}{a}\right)^2\right]+\frac{I\omega^2 r}{cG'_{cx}} \qquad (76)$$

$$a_{31} = \left(\frac{\pi}{b}\right)^4\sqrt{D_{11}D_{22}}\left\{m^2\left(\frac{b}{a}\right)^2\left[\alpha m^2\left(\frac{b}{a}\right)^2+2n^2\beta\right]+\frac{n^4}{\alpha}\right.$$
$$\left.-\frac{\omega^2\left(\frac{b}{\pi}\right)^2}{\sqrt{D_{11}D_{22}}}\left[\left(\frac{mb}{a}\right)^2 I+n^2 I+M\left(\frac{b}{\pi}\right)^2\right]\right\}$$

$$a_{32} = \frac{\pi}{b}\left[m^3\left(\frac{b}{a}\right)^3\alpha V+mn^2\beta V\left(\frac{b}{a}\right)-\frac{I\omega^2}{cG'_{cx}}m\left(\frac{b}{a}\right)\right]$$

$$a_{33} = \frac{\pi}{b}\left[n^3 r\frac{V}{\alpha}+m^2 nr\beta\left(\frac{b}{a}\right)^2 V-\frac{I\omega^2 nr}{cG'_{cx}}\right]$$

COMPRESSION STABILITY

When the loads N_x and N_y reach critical levels, the denominator of equation 74 vanishes. This yields

$$a_{31}+\bar{m}^2N_x+\bar{n}^2N_y = \qquad (77)$$

$$-\frac{\left(a_{33}(a_{12}a_{21}-a_{11}a_{22})+a_{32}(a_{11}a_{23}-a_{13}a_{21})\right)}{a_{12}a_{23}-a_{22}a_{13}} = -a'_{31}$$

Let N_x and N_y be positive for compression; the signs of N_x, N_y are changed in the preceding equation, giving

$$\bar{m}^2N_x+\bar{n}^2N_y = a_{31}+a'_{31}$$

From the last two equations of equation 75 and the preceding, we have

$$\sqrt{D_{11}D_{12}}\left[\left(\frac{m\pi}{a}\right)^2 k_x \left(\frac{\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^2 k_y \left(\frac{\pi}{b}\right)^2\right] = a_{31} + a'_{31}$$

$$\sqrt{D_{11}D_{12}}\left(\frac{\pi}{b}\right)^2\left[\left(\frac{\pi}{a}\right)^2 m^2 k_x + \left(\frac{\pi}{b}\right)^2 n^2 k_y\right] = a_{31} + a'_{31}$$

$$m^2\left(\frac{b}{a}\right)^2 k_x + n^2 k_y = \frac{(a_{31}+a'_{31})\, b^4}{\pi^4\sqrt{D_{11}D_{22}}} = \bar{a}_{31} + \bar{a}'_{31} \qquad (78)$$

where

$$\bar{a}_{31} = \frac{a_{31}\, b^4}{\pi^4\sqrt{D_{11}D_{22}}} \quad \text{and} \quad \bar{a}'_{31} = \frac{a'_{31}\, b^4}{\pi^4\sqrt{D_{11}D_{22}}} \qquad (79)$$

For compressive loading along the x-axis only.

$$k_x = \left(\frac{a}{b}\right)^2 \frac{\bar{a}_{31} + \bar{a}'_{31}}{m^2} \qquad (80)$$

For compressive loading along the y-axis only.

$$k_y = \frac{\bar{a}_{31} + \bar{a}'_{31}}{n^2} \qquad (81)$$

For a constant ratio of $N_y/N_x = \phi$,

$$k_x = \frac{\bar{a}_{31} + \bar{a}'_{31}}{m^2\left(\frac{b}{a}\right)^2 + \phi n^2} \qquad (82)$$

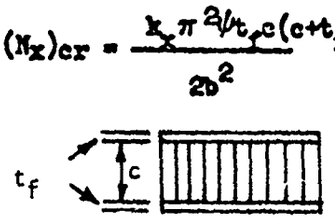
The critical buckling load is obtained (equations 80, 81, and 82) by finding the integers m and n which give minimum values of k_x or k_y. These values for the cases of uniaxial and biaxial compression are shown in figures 29 through 36.

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y}\,/(1-\nu_{xy}\nu_{yx})$$

$$\alpha = \sqrt{E_x/E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\,G_c'(b/c)^2}$$



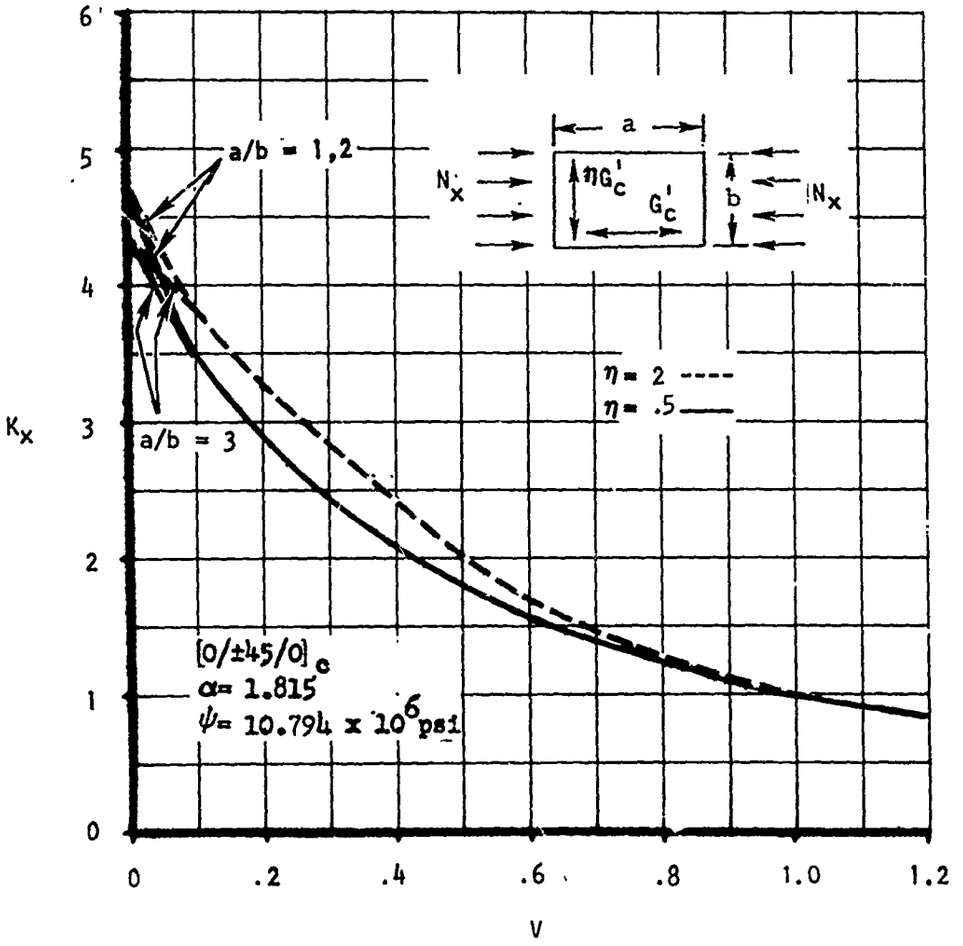


Figure 29. Buckling Coefficient K_x vs V for Simply Supported Flat Sandwich Panels (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y} / \left(1 - \nu_{xy}\nu_{yx}\right)$$

$$\alpha = \sqrt{E_x E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\, G'_c (b/c)^2}$$



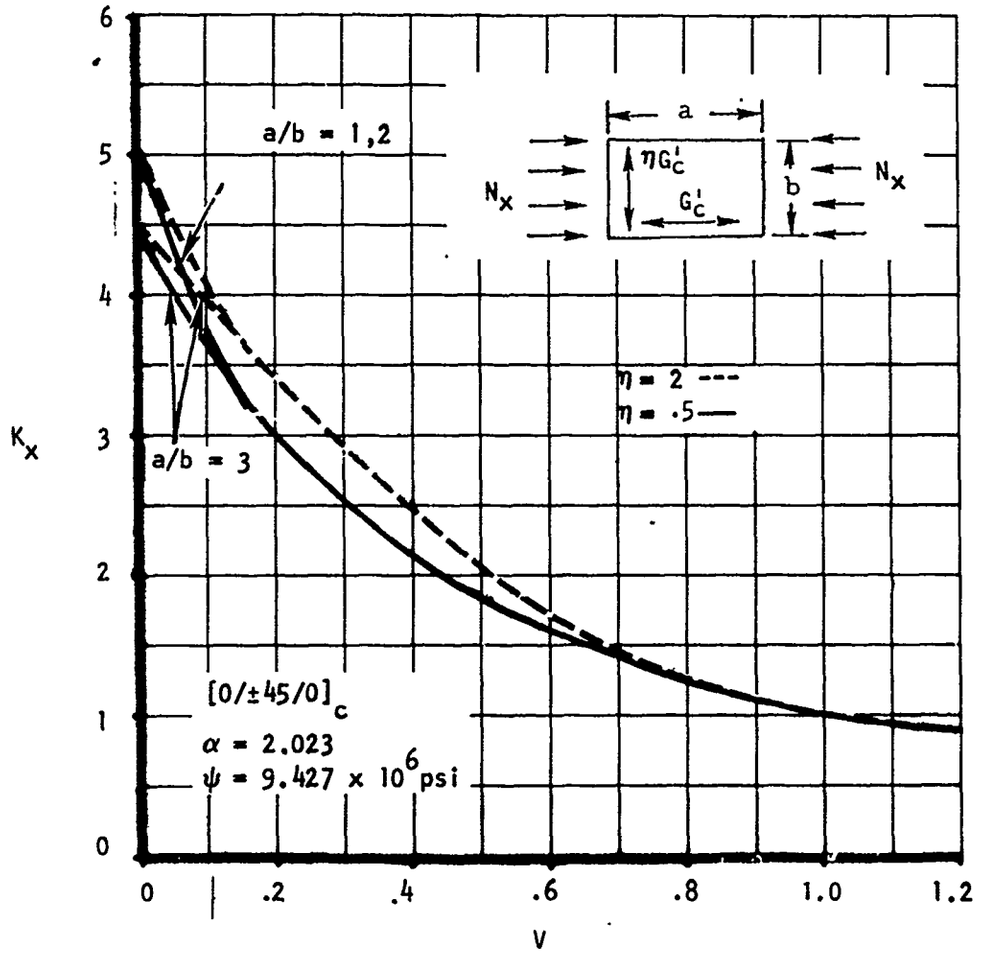


Figure 30. Buckling Coefficient K_x vs F for Simply Supported Flat Sandwich Panels (350° F)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y}\,/(1-\nu_{xy}\nu_{yx})$$

$$\alpha = \sqrt{E_x/E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\,G_c'(b/c)^2}$$



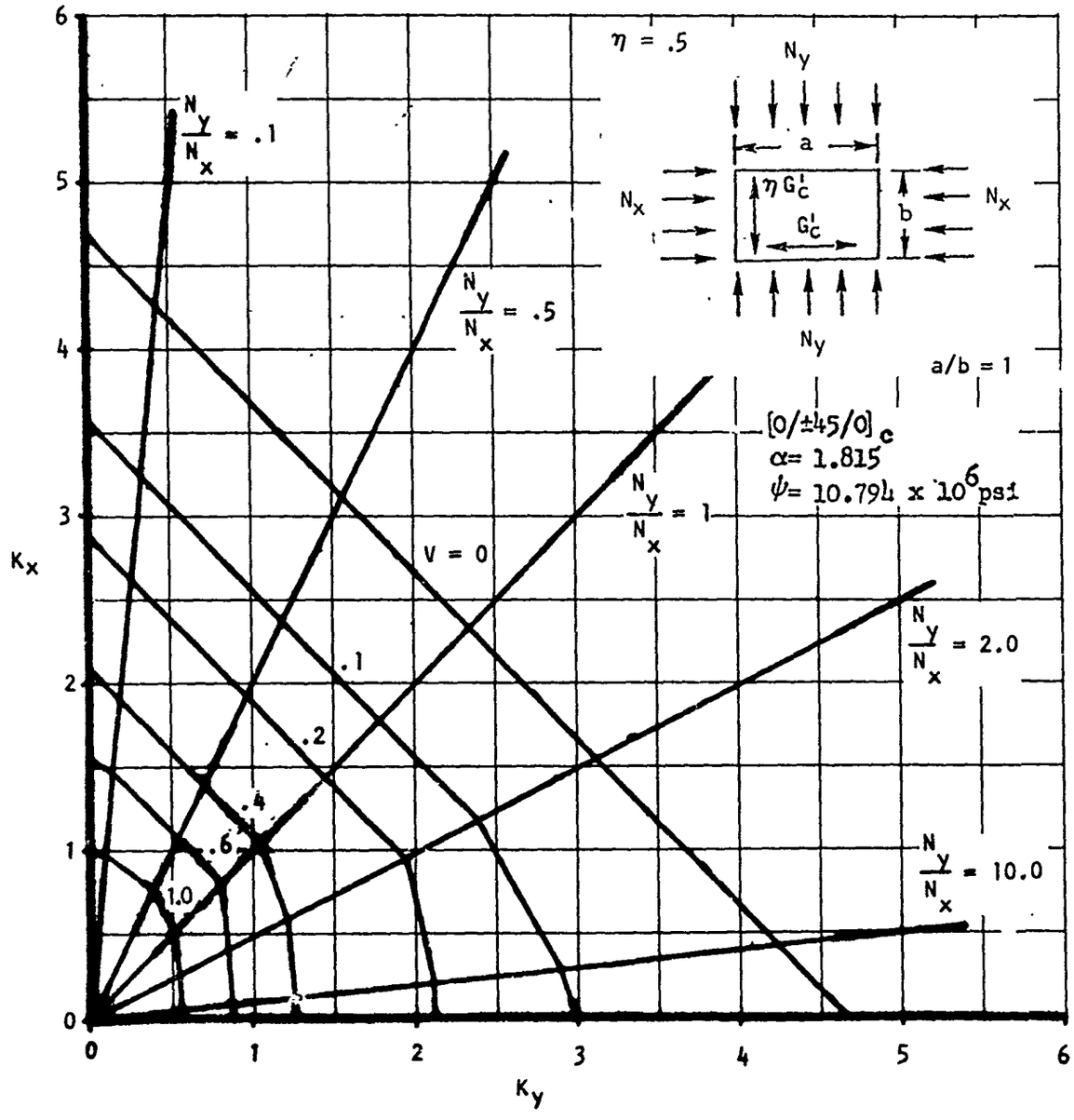


Figure 31. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 1 (RT)

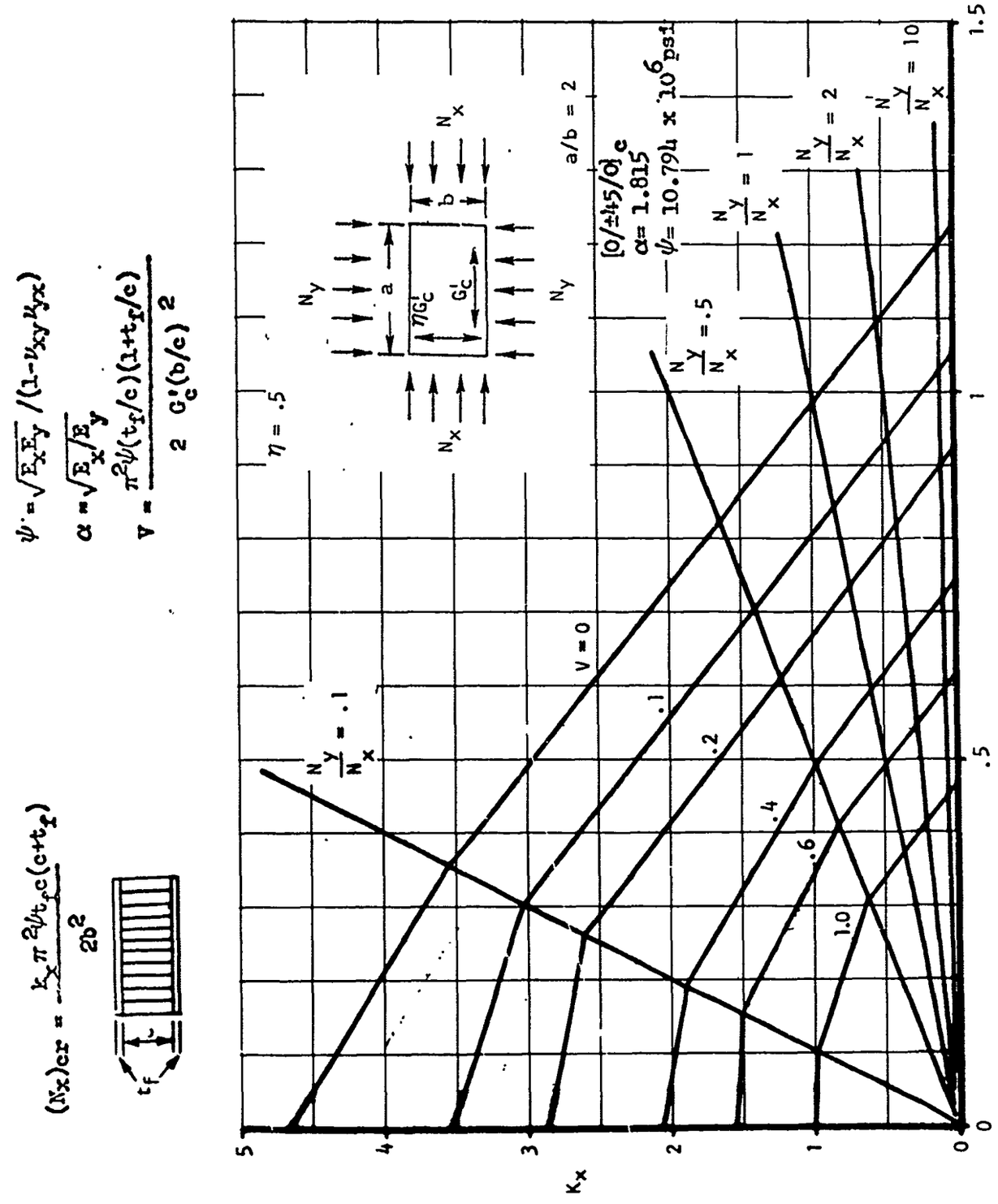


Figure 32. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 2 (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y} / (1-\nu_{xy}\nu_{yx})$$

$$\alpha = \sqrt{E_x/E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\ G_c'(b/c)^2}$$

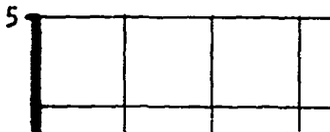


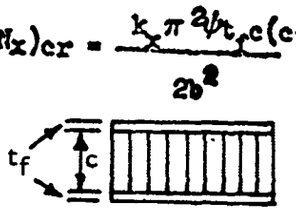
Figure 33. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 3 (RT)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y}/(1-\nu_{xy}\nu_{yx})$$

$$\alpha = \sqrt{E_x/E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\ G_c'(b/c)^2}$$



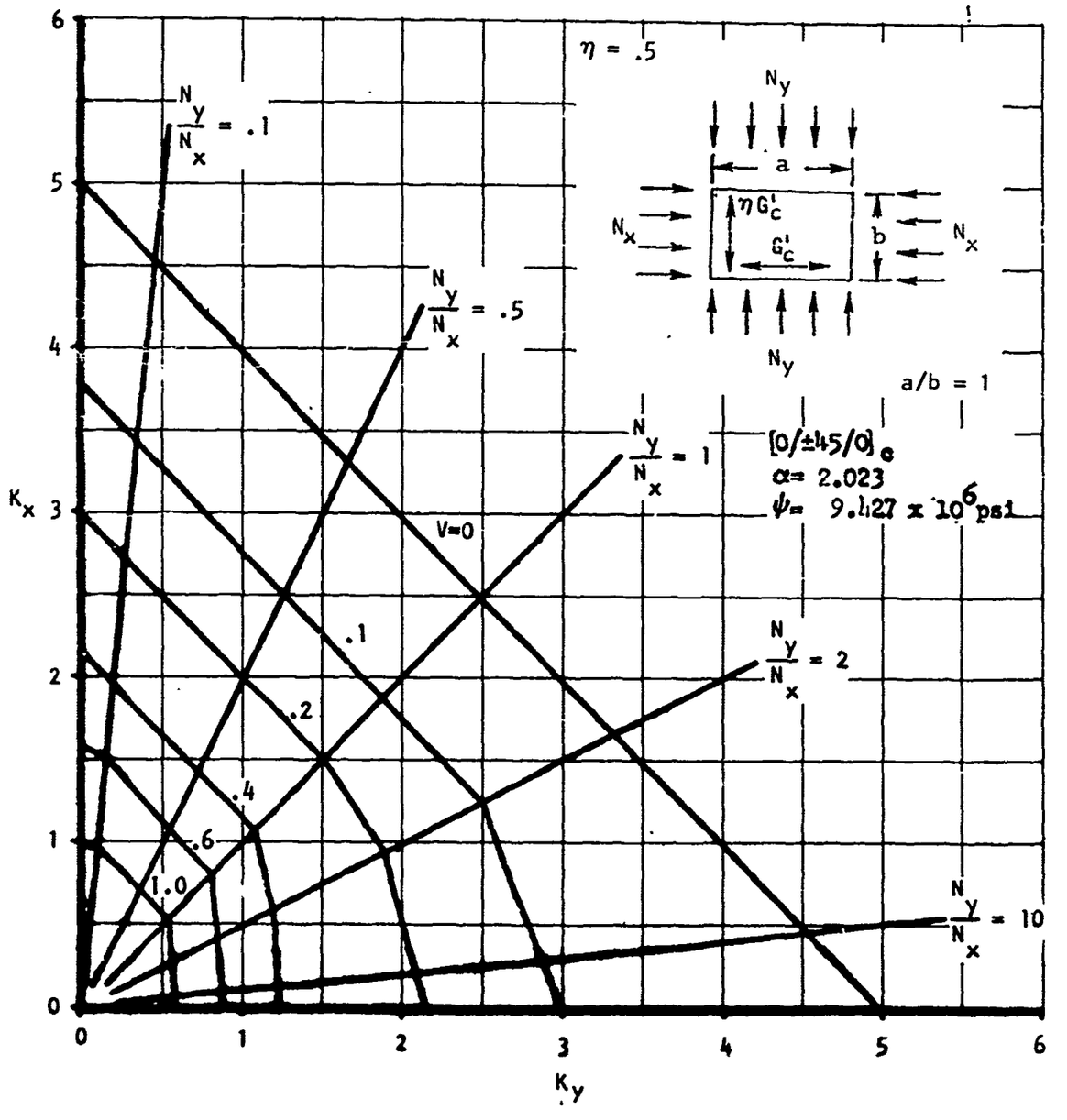


Figure 34. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 1 (350° F)

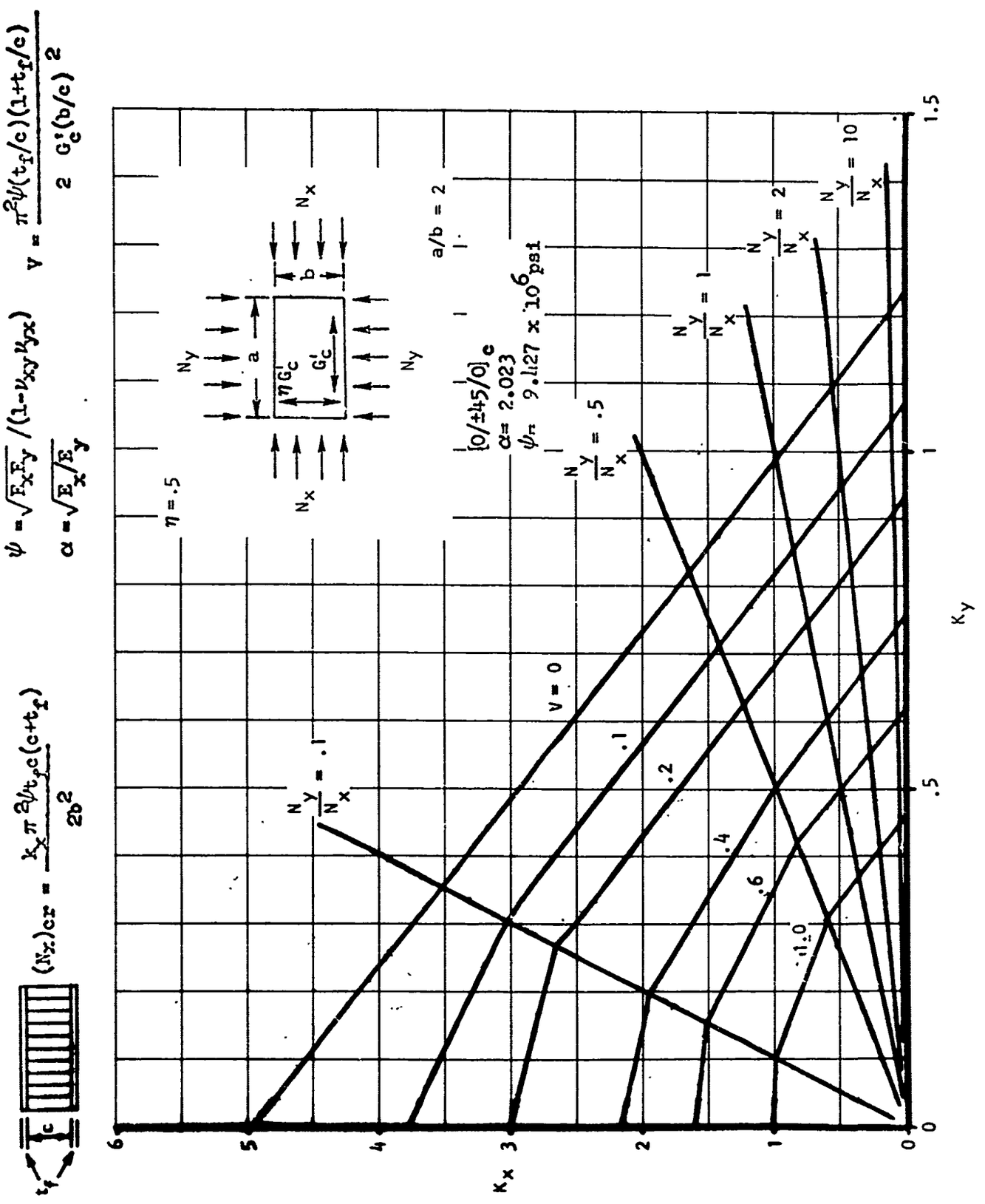


Figure 35. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 2 (350° F)

$$(N_x)_{cr} = \frac{k_x \pi^2 \psi t_f c (c+t_f)}{2b^2}$$

$$\psi = \sqrt{E_x E_y}\,/(1-\nu_{xy}\nu_{yx})$$

$$\alpha = \sqrt{E_x/E_y}$$

$$V = \frac{\pi^2 \psi (t_f/c)(1+t_f/c)}{2\ G_c'(b/c)^2}$$



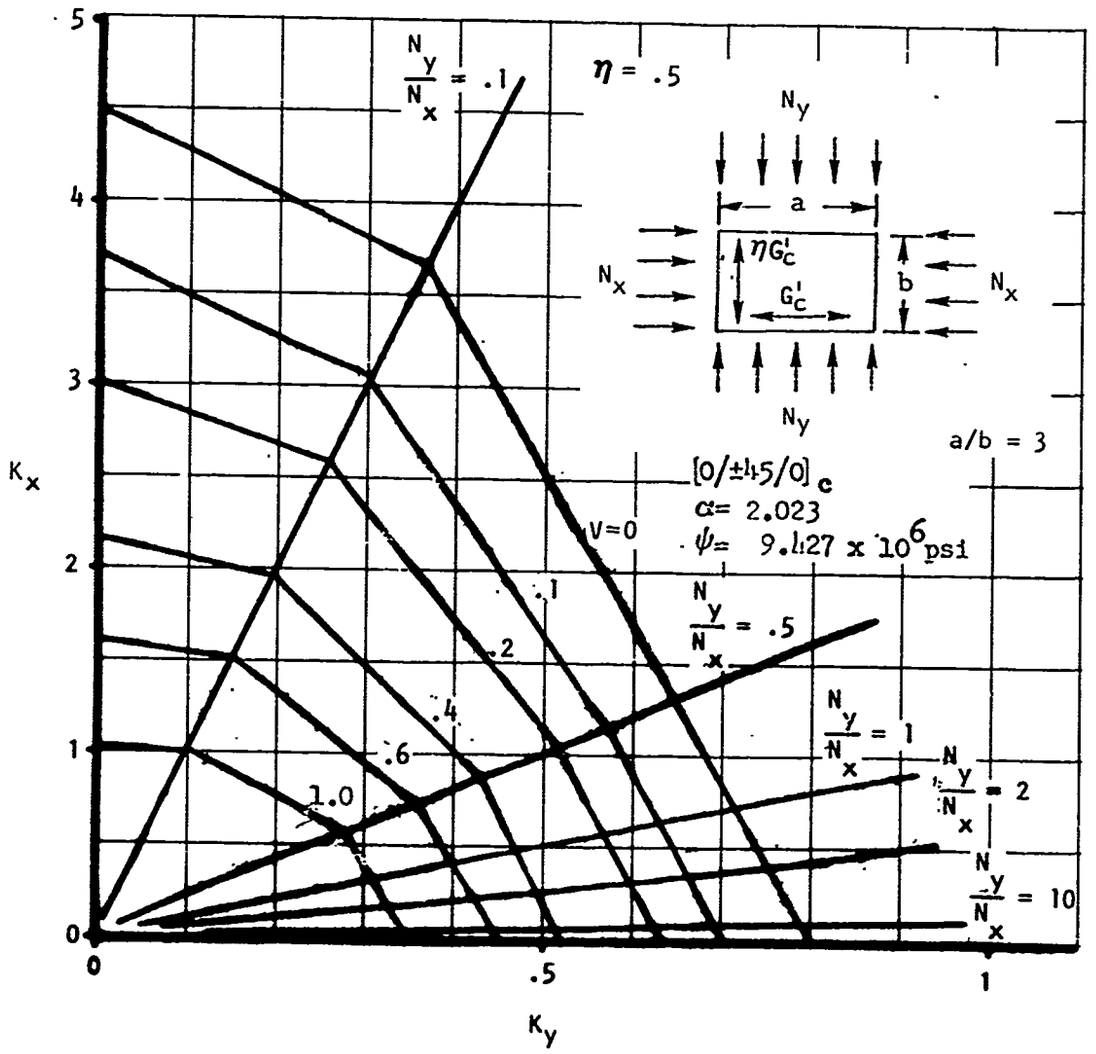


Figure 36. Buckling Coefficients for Biaxially Loaded Simply Supported Flat Sandwich Panels of a/b = 3 (350° F)

In a similar manner, the natural frequencies ω for any m, n mode shape can be obtained by solving equation 78 for given k_x and k_y. As pointed out by previous investigators (reference 3), there are three resonant frequencies for each mode instead of a single resonant frequency for solid plate. Two of these resonances are associated with the two rotations, β_x and β_y, and one is associated with lateral deflection ω. For the cases where the core is isotropic ($G'_{cx} = G'_{cy}$), the three resonant frequencies reduce to two for each mode of vibration.

For filament-reinforced face sheets, the following expressions are invariant with respect to the rotation of the reference axes in the plane of the plate (reference 4).

$$I_1 = \frac{E_x}{1-\nu_{xy}\nu_{yx}}\left(1 + \frac{E_y}{E_x} + 2\nu_{yx}\right) = \frac{E_L}{1-\nu_{LT}\nu_{TL}}\left(1 + \frac{E_T}{E_L} + 2\nu_{TL}\right) \tag{83}$$

where L and T denote the longitudinal and transverse directions of the filaments.

$$I_2 = G_{xy} - \frac{\nu_{yx}E_x}{\left(1-\nu_{xy}\nu_{yx}\right)} = G_{LT} - \frac{\nu_{TL}E_L}{1-\nu_{LT}\nu_{TL}} \tag{84}$$

Since the a_{ij}'s (i,j = 1, 2, 3) depend on the five face-sheet elastic constants through D_{11}, D_{22}, D_{66}, ν_{yx}, and ν_{xy}, the critical values of N_x and N_y depend on these five variables. The buckling constants k_x and k_y are the ratios of N_x and N_y divided by

$$\sqrt{D_{11}D_{22}}\Big/\left(\frac{b}{\pi}\right)^2,$$

and hence are defined by four independent variables. (Only four of the five elastic constants E_x, E_y, ν_{xy}, ν_{yx}, G_{xy} are independent because of the auxiliary relationship $E_x\nu_{yx} = \nu_{xy}E_y$.) By using the invariant relations (equations 83 and 84), k_x and k_y may be expressed in terms of I_1, I_2, and two variables. Let

$$\psi = \sqrt{E_xE_y}\Big/\left(1-\nu_{xy}\nu_{yx}\right) \tag{85}$$

We can express γ and β in terms of α and ψ as

$$\gamma = \frac{1}{2}\left\{\frac{I_1}{\psi} - \alpha - \frac{1}{\alpha} + \frac{2I_2}{\psi}\right\} \tag{86}$$

$$\beta = \frac{3}{2}\left\{\frac{I_1}{\psi} - \alpha - \frac{1}{\alpha}\right\} + \frac{2I_2}{\psi} \tag{87}$$

For a given composite face sheet on a given core with given c, G_{cx}, and G_{cy}, the variation of critical k_x and k_y may be plotted against the properties of the core for different values of α and ψ. This reduces greatly the computation and presentation of the critical values of k_x and k_y for different orientations of the face plates.

A computer program, AC-5, entitled "Honeycomb Panel Stability Under In-plane Biaxial Loading," has been developed to perform the calculations indicated by equations 80 and 81. This program searches and finds the half-wave-length numbers m and n which minimize the buckling load for a given panel geometry and face sheet material. The AC-5 program was utilized to generate the stability curves for the $[0_2/\pm 45]_C$ laminate orientations used for the structural elements tested in this program and reported in volume II.

For the Narmco 5505 system, the values of the material invariants used were

$$\left.\begin{aligned} I_1 &= 33.88 \text{ Msi} \\ I_2 &= 0.128 \text{ Msi} \end{aligned}\right\} \quad \text{for room temperature}$$

$$\left.\begin{aligned} I_1 &= 31.54 \text{ Msi} \\ I_2 &= 0.066 \text{ Msi} \end{aligned}\right\} \quad \text{for } 350^\circ \text{ F}$$

Figures 29 and 30 give the uniaxial buckling loads at room temperature and 350° F, respectively, for the $[0_2/\pm 45]_C$ orientation. Figures 31 through 36 give the buckling loads for the same orientation at room temperature and 350° F for various combinations of longitudinal and transverse in-plane loads and aspect ratios.

UNIFORMLY DISTRIBUTED PRESSURE AND EDGE COMPRESSION

We now consider that the previously discussed panel is subjected to a uniform lateral load q as shown in figure 37. The pressure q can be expanded into the following Fourier sine series:

$$q = \sum_m \sum_n q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{88}$$

where

$q_{mn} = 16q/\pi^2 mn$ for odd values of m and n

$q_{mn} = 0$ for m or n even

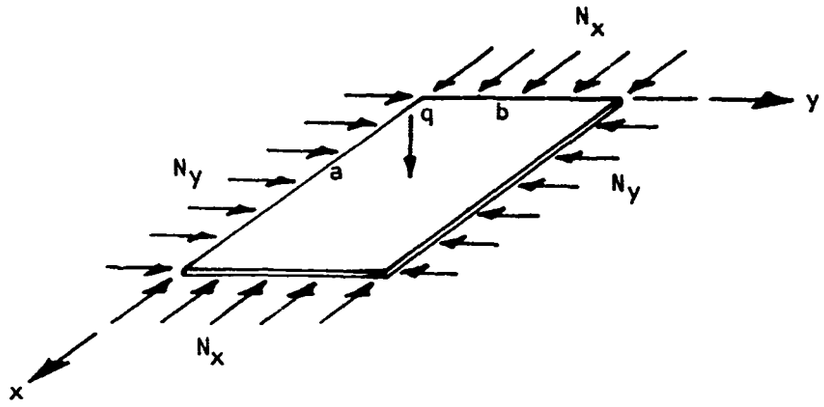


Figure 37. Sandwich Panel Subjected to Normal Load and Edge Compression

In this case, the loading is static and the radial velocity is zero, i.e., $\omega = 0$; thus, the panel deflection is expressed as

$$W = \sum_{m=1,3}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{89}$$

and we obtain from equations 74 and 88 that A_{mn} takes the form, with N_x and N_y taken to be positive for compression:

$$A_{mn} = \frac{16\, q\, b^4\, \bar{A}_{mn}}{\pi^6 mn \sqrt{D_{11}D_{22}}} \tag{90}$$

where

$$\bar{A}_{mn} = \frac{\left(\bar{a}_{12}\bar{a}_{23} - \bar{a}_{22}\bar{a}_{13}\right)}{\left[\bar{a}_{31} + R_x k_x \left(\frac{mb}{a}\right)^2 + R_y k_y n^2\right]\bar{a}_{22}\left(\bar{a}_{13}\bar{a}_{12}\bar{a}_{23}\right) + \bar{a}_{32}\left(\bar{a}_{13}\bar{a}_{21} - \bar{a}_{11}\bar{a}_{23}\right)} \cdots$$

$$\cdots + \bar{a}_{33}\left(\bar{a}_{11}\bar{a}_{22} - \bar{a}_{12}\bar{a}_{21}\right) \tag{91}$$

and

$$\left.\begin{aligned}
\bar{a}_{11} &= a_{11}\Big/\left(\frac{\pi}{b}\right)^3\sqrt{D_{11}D_{22}} \\
\bar{a}_{12} &= a_{12} \\
\bar{a}_{13} &= -a_{13} \\
\bar{a}_{21} &= a_{21}\Big/\left(\frac{\pi}{b}\right)^3\sqrt{D_{11}D_{22}} \\
\bar{a}_{22} &= a_{22} \\
\bar{a}_{23} &= -a_{23} \\
\bar{a}_{31} &= -a_{31}\Big/\left(\frac{\pi}{b}\right)^4\sqrt{D_{11}D_{22}}
\end{aligned}\right\} \tag{92}$$

From equations 53, 62, and 63, we find

$$M_x = D_{11}\left(\frac{\partial\beta_x}{\partial x} + \nu_{yx}\frac{\partial\beta_y}{\partial y}\right)$$

$$= \sum_m\sum_n - D_{11}\left\{\left[\frac{B_{mn}\left(\frac{m\pi}{a}\right)}{cG_{cx}'} - \left(\frac{m\pi}{a}\right)^2 A_{mn}\right]\right.$$

$$\left. + \nu_{yx}\left[\frac{C_{mn}\left(\frac{n\pi}{b}\right)}{cG_{cy}'} - \left(\frac{n\pi}{b}\right)^2 A_{mn}\right]\right\}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{93}$$

Equations 72 and 73 can be recast in the form

$$C_{mn} = A_{mn}\left(\frac{\pi}{b}\right)^3\sqrt{D_{11}D_{22}}\;\frac{\left(\bar{a}_{11}\bar{a}_{22} - \bar{a}_{12}\bar{a}_{21}\right)}{\left(\bar{a}_{22}\bar{a}_{13} - \bar{a}_{12}\bar{a}_{23}\right)} \tag{94}$$

$$= \lambda_2\left(\frac{\pi}{b}\right)^3\sqrt{D_{11}D_{22}}\;A_{mn}$$

and

$$B_{mn} = A_{mn}\left(\frac{\pi}{b}\right)^3 \sqrt{D_{11}D_{22}}\ \frac{\left(\bar{a}_{11}\bar{a}_{23} - \bar{a}_{13}\bar{a}_{21}\right)}{\left(\bar{a}_{22}\bar{a}_{13} - \bar{a}_{12}\bar{a}_{23}\right)} \tag{95}$$

$$= \lambda_1 \left(\frac{\pi}{b}\right)^3 \sqrt{D_{11}D_{22}}\ A_{mn}$$

$$\bar{a}_{32} = -a_{32}/(\pi/b)$$

$$\bar{a}_{33} = a_{33}/(\pi/b$$

$$R_x = N_x/N_{x_{cr}}$$

$$R_y = N_y/N_{y_{cr}}$$

we obtain the following desired form for M_x by combining equations 90, 93, 94, and 95:

$$M_x = -\frac{\alpha 16\ q\ b^2}{\pi 4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\bar{A}_{mn}}{mn} \left\{ \left[m\frac{b}{a} \lambda_1 V - \left(\frac{mb}{a}\right)^2 \right] \right. \tag{96}$$

$$\left. + \nu_{yx} \left[rVh\lambda_2 - n^2 \right] \right\} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

In a similar manner, we find that

$$M_y = -\frac{16\,q\,b^2}{\alpha\pi^4}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\left\{\left[n\lambda_2 rV - n^2\right] + \nu_{xy}\left[\lambda_1 m\frac{b}{a} - \left(\frac{mb}{a}\right)^2\right]\right\}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{97}$$

$$M_{xy} = \frac{16\,q\,b^2\gamma}{\pi^4}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\bar{A}_{mn}\left\{\frac{\lambda_1 V}{m} + \frac{\lambda_2 rV(b/a)}{n} - 2\,(b/a)\right\}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{98}$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$$

$$= -\frac{16\,q\,b}{\pi^3}\sum_{m=1,3}^{\infty}\sum_{n=1,3}^{\infty}\bar{A}_{mn}\left\{\left(\frac{b}{a}\right)\frac{1}{n}\left[\left(\beta-2\gamma\right)\left(n\lambda_2 rV - n^2\right) + \alpha\left(\lambda_1 m\frac{b}{a}V - \frac{m^2b^2}{a^2}\right)\right] + \gamma n\left[\frac{\lambda_1 V}{m} + \lambda_2\frac{rV(b/a)}{n} - 2\left(\frac{b}{a}\right)\right]\right\}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{99}$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{yx}}{\partial x}$$

$$= \frac{-16\, q\, b}{\pi^3} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \bar{A}_{mn} \left\{ \gamma \frac{mb}{a} \left[\frac{\lambda_1 V}{m} + \lambda_2 \frac{rV\,(b/a)}{n} - 2\left(\frac{b}{a}\right) \right] + \frac{1}{m} \left[\frac{\left(n\lambda_2\, rV - n^2\right)}{\alpha} + (\beta - 2\gamma) \left(\lambda_1\, m \frac{b}{a} V - m^2 \frac{b^2}{a^2} \right) \right] \right\} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \tag{100}$$

where the following auxiliary relationships were used to reduce the preceding equations to their final form:

$$\left.\begin{aligned} \nu_{xy} &= \nu_{yx} \frac{E_x}{E_y} = \nu_{yx} \alpha^2 \\ \nu_{yx} \alpha &= \beta - 2\gamma \end{aligned}\right\} \tag{101}$$

The stresses in the panel are then determined from the relationship

$$\left.\begin{aligned} \tau_{xz} &= Q_x/c, \quad \tau_{yz} = Q_y/c \\ \sigma_x &= \pm \frac{M_x}{t_f\left(c+t_f\right)} - \frac{N_x}{2t_f} \\ \sigma_y &= \pm \frac{M_y}{t_f\left(c+t_f\right)} - \frac{N_y}{2t_f} \\ \tau_{xy} &= \pm \frac{M_{xy}}{t_f\left(c+t_f\right)} \end{aligned}\right\} \tag{102}$$

and, as mentioned before, the in-plane forces N_y and N_x are taken to be positive if they are compressive.

A computer program AC-7 has been developed to perform the calculations to obtain the normal deflections and internal moments and shears of simply supported honeycomb sandwich panels with orthotropic core and faces subjected to normal pressure and in-plane forces. Tables I and II contain data obtained from AC-7 for the structural elements tested in this program and reported in

volume II of this report. Figure 38 is a plot of the maximum normal deflection for panels with face sheets composed of $[0_2/\pm45]_C$ laminates and no in-plane forces. These data were obtained from AC-7.

TABLE I. INTERNAL MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED PANELS WITH NO IN-PLANE LOADS*

a/b	Temperature	M_x/qb^2 x/a = 0.5 y/b = 0.5	M_y/qb^2 x/a = 0.5 y/b = 0.5	Q_x/qb x/a = 0.0 y/b = 0.5	Q_y/qb x/a = 0.5 y/b = 0.0
1.0	RT	0.073	0.027	0.396	0.262
	350° F	0.072	0.026	0.396	0.254
2.0	RT	0.094	0.077	0.485	0.408
	350° F	0.104	0.071	0.512	0.395
3.0	RT	0.092	0.104	0.496	0.461
	350° F	0.107	0.099	0.535	0.451

* Laminate orientation - $[0_2/\pm45]_C$

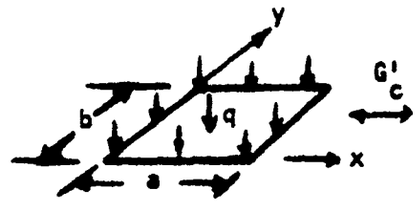
Material - Narmco 5505

$$\frac{G'_{cx}}{G'_{cy}} = \frac{DQ_x}{DQ_y} = 2.0$$

$$V = \frac{\pi^2 \left(t_f/c\right)\left(1+t_f/c\right)\psi}{2G'_c\left(b/c\right)^2}$$

$$\xi = \frac{t_f c\left(c+t_f\right)\psi}{2}$$

$$\psi = \sqrt{E_x E_y}/\left(1-\nu_{xy}\nu_{yx}\right)$$



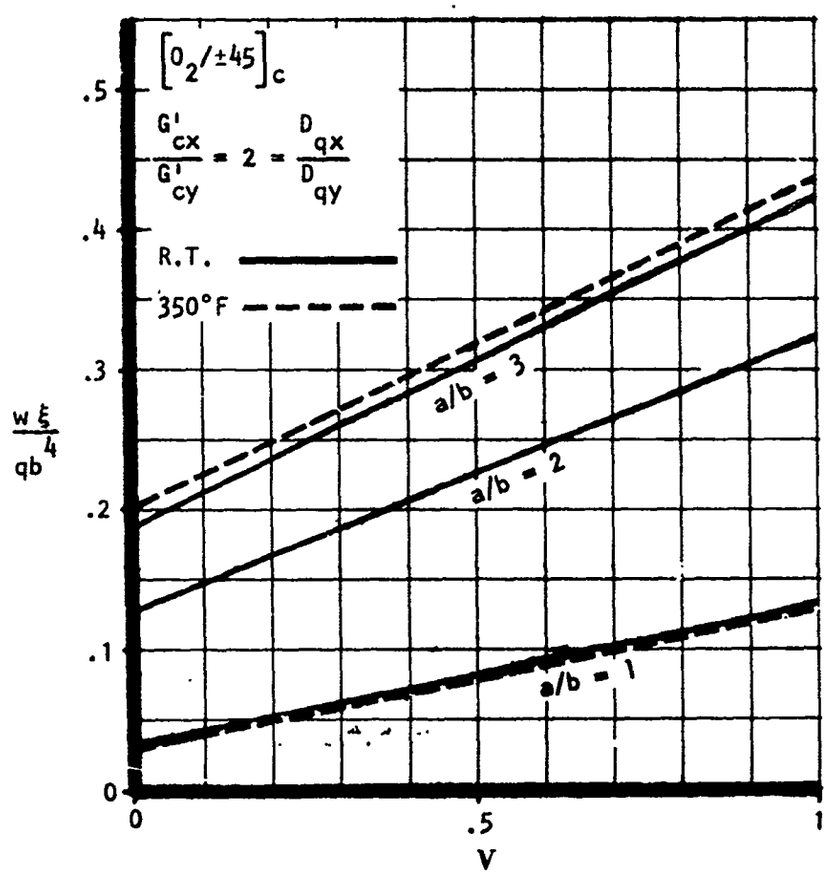


Figure 38. Parameter $w\,\xi/qb^4$ vs V for Simply Supported Honeycomb Sandwich Panels

TABLE II. MAXIMUM NORMAL DEFLECTION AND INTERNAL MOMENTS AND SHEARS FOR SIMPLY SUPPORTED PANELS* WITH $N_x/(N_x)_{cr} = 0.333$

a/b	Temperature	$W\xi/qb^4$ x/a = 0.5 y/b = 0.5	M_x/qb^2 x/a = 0.5 y/b = 0.5	M_y/qb^2 x/a = 0.5 y/b = 0.5	Q_x/qb x/a = 0.0 y/b = 0.5	$Q_y/2b$ x/a = 0.5 y/b = 0.0
1.0	RT	0.00581	0.107	0.044	0.562	0.356
	350° F	0.00547	0.111	0.040	0.573	0.342
2.0	RT	0.0290	0.121	0.149	0.850	0.594
	350° F	0.0308	0.148	0.145	0.864	0.597

*Laminate orientation - $[0_2/\pm45]_C$

Material - Narmco 5505

$$\frac{G'_{cx}}{G'_{cy}} = \frac{DQ_x}{DQ_y} = 2.0$$

$$\xi = \sqrt{D_{11}\, D_{22}}$$

SHEAR STABILITY OF FLAT SANDWICH PANELS

A computerized method has been developed to predict the shear stability allowable of flat sandwich panels with orthotropic face sheets and core. The face sheets are assumed to be similar, and the shear loading is in the plane of the panel. The analytical development is given in reference 5, and only the final governing equations are presented, along with the listing of the computer program and a sample problem solution.

Consider the sandwich panel to lie in the X-Y plane with the Z-axis perpendicular to the face sheets, as shown in figure 39. The critical shear buckling load is given by

$$(N_{xy})_{cr} = K_s \frac{\pi^2 \psi t_f (c+t_f)^2}{2b^2} = (N_s)_{cr} \qquad (103)$$

where

$$\psi = \frac{\sqrt{E_x E_y}}{1-\nu_{xy}\nu_{yx}}$$

and the other terms are defined in the figure.

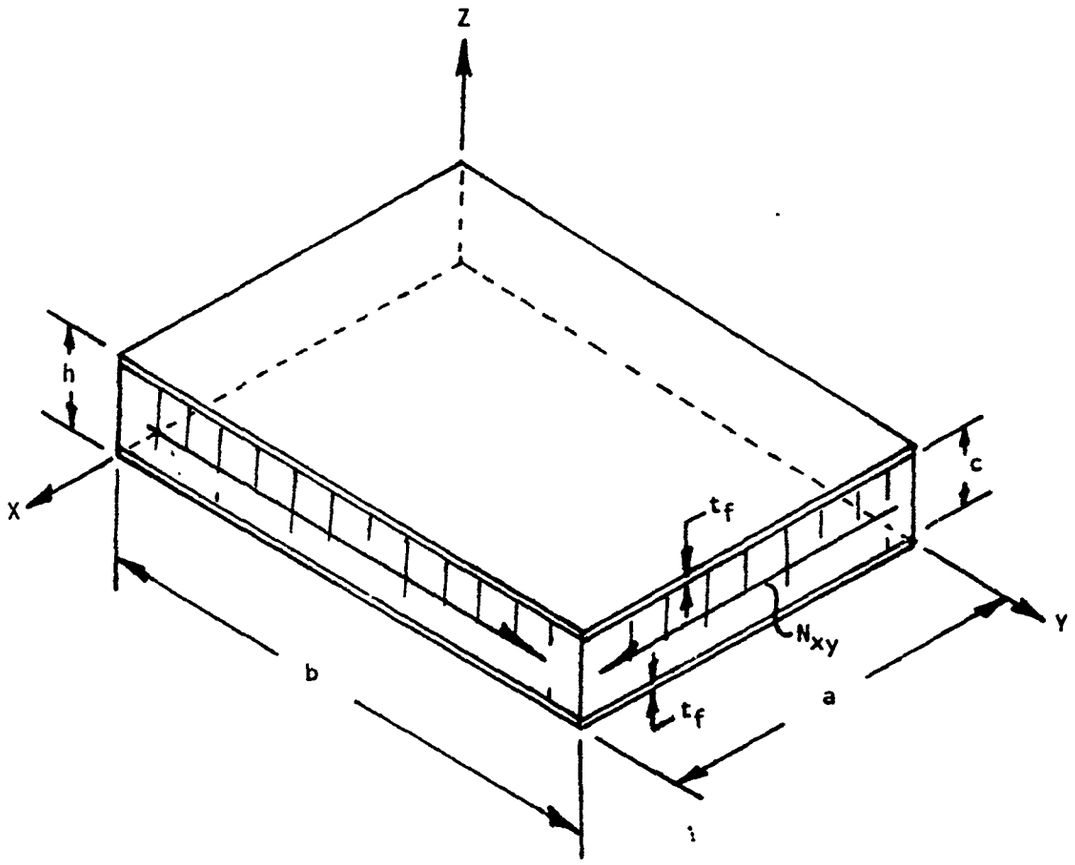


Figure 39. Typical Honeycomb Sandwich Panel Subjected to In-plane Shear Loading

K_s is a constant that is found by the solution of the following governing equation that is derived using energy principles and is

$$\left| \frac{1}{K_s} A_{mn} C_{mn} + \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} H_{mnrs} C_{rs} \right| = 0 \qquad (104)$$

where

$$A_{mn} = (2V_{mni} + V_{mn}) \left(\frac{b}{a}\right)^3$$

$$H_{mnrs} = \begin{cases} \dfrac{32\, mnrs}{\pi^2 (m^2-r^2)(n^2-s^2)}, & \text{if } m + r \text{ and } n + s \text{ are odd} \\ 0, & \text{otherwise.} \end{cases}$$

$$V_{mni} = \frac{1}{6} \left(\frac{t_f}{c+t_f}\right)^2$$

$$V_{mn} = \frac{K_{mn} + \left(\dfrac{n^2 a^2}{b^2} S_x + m^2 S_y\right) F_{mn}}{1 + L_{mn} S_x + S_x S_y F_{mn}}$$

$$F_{mn} = 1 - \beta^2 + \gamma K_{mn}$$

$$K_{mn} = \alpha m^4 + 2\beta \frac{m^2 n^2 a^2}{b^2} + \frac{n^4 a^4}{\alpha b^4}$$

$$L_{mn} = \alpha m^2 + \gamma \frac{n^2 a^2}{b^2} + r \left(\frac{n^2 a^2}{\alpha b^2} + \gamma m^2 \right)$$

$$\alpha = \sqrt{\frac{E_x}{E_y}}$$

$$\beta = \alpha \nu_{yx} + 2\gamma$$

$$\gamma = G_{xy}/\psi$$

$$r = \frac{G'_{cx}}{G'_{cy}}$$

$$S_x = \frac{c\pi^2 4t_f}{2a^2 G'_{cx}}$$

$$S_y = rS_x$$

G'_{cx}, G'_{cy} = core shear moduli in the x and y direction, respectively. Using the contracted notation that

$$i = n + q(m - 1)$$

$$j = s + q(r - 1)$$

where q is the limit of m, n, r, and s, the resulting governing equation becomes

$$\left| \overline{K} A_i C_i + \sum_{j=1}^{q^2} H_{ij} C_j \right| = 0 \tag{105}$$

where

$$\overline{K} = \frac{1}{K_s}$$

Dividing through by A_i and rearranging terms results in

$$\overline{K} C_i = - \sum_{j=1}^{q^2} \frac{H_{ij} C_j}{A_i} \tag{106}$$

which in matrix notation becomes

$$\left| \left[B\right] \left|C\right| - \overline{K} \left|C\right| \right| = 0$$

where

$$B_{ij} = - \frac{H_{ij}}{A_i} \tag{107}$$

It is noticed that the last equation is in the form of the standard eigenvalue problem. For this type of problem, a modified matrix iteration process (reference 7) will yield the highest eigenvalue, which means that we have the lowest value of K_s since

$$K_s = \frac{1}{\overline{K}}$$

To facilitate the solution for $\overline{K}$, it is pointed out in reference 5 that the matrix [B] can be separated into even and odd terms of C_{mn}, i.e., [B] even for m + n even and [B] odd for m + n odd. Then, [B] even and [B] odd can each be solved independently for K_s even and K_s odd, and the minimum of these two values would be the same value, K_s, that would result if the original [B] matrix were utilized to find K_s. This procedure greatly reduces computing time.

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SECTION V

STIFFENED SKIN CONSTRUCTION

CONSTITUTIVE RELATIONSHIPS

The analysis in this section for an N-ply flat sheet stiffened with Z or hat stiffeners parallels the work performed by Lackman and Ault (reference 6), but is extended to account for a heterogeneous sheet rather than the homogeneous sheet treated by these authors. We now seek to analyze the typical stiffened sheet shown in figure 40 when the coordinate origin is located in the midplane of the sheet.

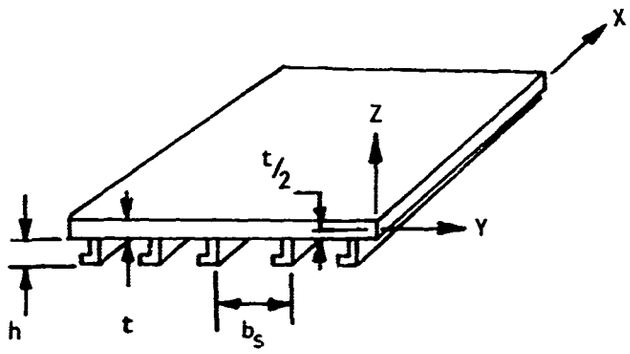


Figure 40. Typical Stiffened Sheet

From equation 5 we have the following relationship between the stress and displacement components of the i^{th} ply of the stiffened sheet shown in figure 40:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = \left[\bar{Q}_{ij}\right]_i \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} - z\dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial v_o}{\partial y} - z\dfrac{\partial^2 w}{\partial y^2} \\ \dfrac{\partial v_o}{\partial x} + \dfrac{\partial u_o}{\partial y} - 2z\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

And if we further consider each sheet to be composed of ply orientations which are symmetric relative to its midsurface, then we have from equation 7 the following relationship between the sheet in-plane forces:

$$\begin{Bmatrix} N_x^f \\ N_y^f \\ N_{xy}^f \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \partial u_o/\partial x \\ \partial v_o/\partial y \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix}$$

where

$$A_{ij} = \sum_{1}^{N} \left(\bar{Q}^i_{ij}\right) \left(h_{i+1} - h_i\right)$$

and $\bar{Q}_{ij}$ is defined in equation 2.

The axial force per unit width, N_x, along the X axis of the stiffened sheet is obtained from the relationship

$$b_s N_x = \int_{-b_s/2}^{b_s/2} \left\{ \underbrace{\int_{-(h_s + t/2)}^{-t/2} \sigma_x dz}_{\text{stiffener}} + \int_{-t/2}^{t/2} \sigma_x dz \right\} dy = N_x^s + b_x N_x^f \qquad (108)$$

when the N_x^f and N_x^s denote the force per unit width carried by face and stiffener, respectively. If we now consider N_x^s is of the form

$$N_x^s = K_1 \frac{\partial u_o}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2} \qquad (109)$$

then, by combining equations 109 and 7 with equation 108, we obtain, after integrating,

$$N_x = \left(A_{11} + \frac{K_1}{b_s}\right)\frac{\partial u_o}{\partial x} + A_{12}\frac{\partial v_o}{\partial y} + A_{16}\left(\frac{\partial v_o}{\partial x} + \frac{\partial u_o}{\partial y}\right)$$

$$+ \frac{K_2}{b_s}\frac{\partial^2 w}{\partial x^2} \qquad (110)$$

Noting that in the Y direction only the sheet carries the N_y and N_{yx} in-plane loads, we obtain from equation 7

$$N_y = N_y^f = A_{12}\frac{\partial u_o}{\partial x} + A_{22}\frac{\partial v_o}{\partial y} + A_{26}\left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\right)$$

$$N_{xy} = N_{xy}^f = A_{16}\frac{\partial u_o}{\partial x} + A_{26}\frac{\partial v_o}{\partial y} + A_{66}\left(\frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x}\right) \qquad (111)$$

From equation 7b, we have the following relationship between the moment components of the sheet and the curvature of the stiffened sheet, namely:

$$\begin{Bmatrix} M_x^f \\ M_y^f \\ M_{xy}^f \end{Bmatrix} = - \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial x^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

where

$$D_{ij} = \sum_{i=1}^{N} \frac{\bar{Q}_{ij}^{(i)}}{3}\left(h_{i+1}^3 - h_i^3\right)$$

If we further consider the bending moment component in the X direction carried by the stiffener to be of the form

$$M_x^s = C_1 \frac{\partial u_o}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2} \tag{112}$$

then we find

$$b_s M_x = \int_{-b_s/2}^{b_s/2} \left\{ \underbrace{\int_{-(h_s + t/2)}^{t/2} \sigma_x z dz}_{\text{stiffener}} + \int_{-t/2}^{t/2} \sigma_x z dz \right\} dy$$

$$= M_x^s + b_s M_x^f \tag{113}$$

Thus, the bending moment components for the stiffened sheet become

$$M_x = \frac{1}{b_s} \left(C_1 \frac{\partial u_o}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2} \right) - \left(D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2} + 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \right) \tag{114}$$

$$M_y = - \left(D_{12} \frac{\partial^2 w}{\partial x^2} + D_{22} \frac{\partial^2 w}{\partial y^2} + 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \right) = M_y^f \tag{115}$$

$$M_{xy} = - \left(D_{16} \frac{\partial^2 w}{\partial x^2} + D_{26} \frac{\partial^2 w}{\partial y^2} + 2D_{66} \frac{\partial^2 w}{\partial x \partial y} \right) = M_{xy}^f \tag{116}$$

If we invoke the condition that no additional in-plane forces N_x, N_y, or N_{xy} are introduced by the plate displacement components u_o, v_o, and w, then by setting the in-plane force components equal to zero, we find

$$\frac{\partial v_o}{\partial y} = \frac{K_2' \left[A_{12} - \frac{A_{16}A_{26}}{A_{66}}\right] \frac{\partial^2 w}{\partial x^2}}{\left[K_1' + A_{11} - \frac{A_{16}^2}{A_{66}}\right]\left[A_{22} - \frac{A_{26}^2}{A_{66}}\right] - \left[A_{12} - A_{16}\frac{A_{26}}{A_{66}}\right]^2}$$

$$= \overline{\alpha} \frac{\partial^2 w}{\partial x^2} \tag{117}$$

$$\frac{\partial u_o}{\partial x} = \frac{K_2' \left[A_{22} - \frac{A_{26}^2}{A_{66}}\right] \frac{\partial^2 w}{\partial x^2}}{-\left[K_1' + A_{11} - \frac{A_{16}^2}{A_{66}}\right]\left[A_{22} - \frac{A_{26}^2}{A_{66}}\right] + \left[A_{12} - A_{16}\frac{A_{26}}{A_{66}}\right]^2}$$

$$= \overline{\beta} \frac{\partial^2 w}{\partial x^2} \tag{118}$$

where

$$K_2' = K_2/b_s$$

$$K_1' = K_1/b_s$$

$$\overline{\alpha} = -\Delta \left[A_{12} - \frac{A_{16}A_{26}}{A_{66}}\right]$$

$$\overline{\beta} = \Delta \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right]$$

$$\Delta = K_2' \Big/ \left\{ \left[K_1' + A_{11} - A_{16}^2/A_{66} \right] \left[A_{22} - \frac{A_{26}^2}{A_{66}} \right] + \left[A_{12} - \frac{A_{16}A_{26}}{A_{66}} \right]^2 \right\}$$

Substitution of equation 118 into 114 yields

$$M_x = \left(\frac{C_1 \overline{\beta}}{b_s} + \frac{C_2}{b_s} - D_{11} \right) \frac{\partial^2 w}{\partial x^2} - D_{12} \frac{\partial^2 w}{\partial y^2} - 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \qquad (119)$$

If the plate is now subjected to an in-plane compression force $\overline{N}_x$ and if we neglect the changes in the in-plane forces due to the plate's buckling displacements, the governing equilibrium equation for the plate shown in figure 18 is obtained from equation 26 as

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \overline{N}_x \frac{\partial^2 w}{\partial x^2} \qquad (120)$$

Substitution of equations 115, 116, and 119 into equation 120 yields

$$D_1 \frac{\partial^4 w}{\partial x^4} + 4 D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2 \left(D_{12} + 2 D_{66}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2}$$

$$+ 4D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} = -\bar{N}_x \frac{\partial^2 w}{\partial x^2} \quad (121)$$

where

$$D_1 = \left(-C_1' \bar{\beta} - C_2' + D_{11}\right)$$

$$C_1' = C_1/b_s$$

$$C_2' = C_2/b_s$$

By comparing equations 121 and 27c, it can be seen that the solution to any anisotropic, laminated, stiffened plate can be obtained from known solutions of equation 27c simply by substituting D_1 for D_{11}. For example, for the case of a simply supported wide-column construction, we take w in the form

$$w = A \sin \frac{\pi x}{L} \quad (122)$$

and we find by substituting the chosen form of w into equation 12 that the critical buckling load is

$$\left(\bar{N}_x\right)_{cr} = \frac{\pi^2 D_1}{L^2} \quad (123)$$

i.e., the classical solution has been modified by replacing D_{11} with D_1.

ZEE STIFFENERS

To utilize equation 121 for Z-stiffened sheets, the values of C_1', C_2', K_1', and K_2' must be established. Thus, we now consider the typical Z-stiffener shown in figure 41. Referring to equation 108, we see that

$$N_x^s = \int_{-\left(\frac{t_s}{2}+f_s\right)}^{-t_s/2}\int_{-\left(h_s+\frac{t}{2}\right)}^{-\left(h_s+t/2-t_{s_1}\right)}\sigma_x\,dzdy + \int_{-t_s/2}^{t_s/2}\int_{-\left(h_s+t/2\right)}^{-t/2}\sigma_x\,dzdy + \int_{t_s/2}^{\frac{t_s}{2}+d_s}\int_{-\left(t/2+t_{s_1}\right)}^{-t/2}\sigma_x dzdy \tag{124}$$

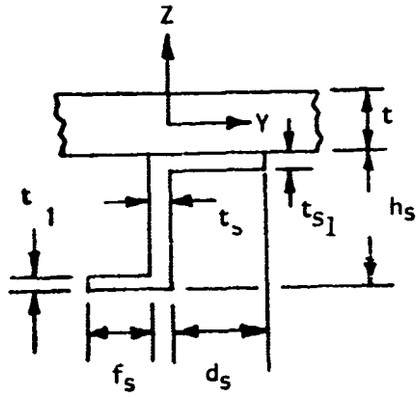


Figure 41. Typical Zee Stiffener

With the assumption that the stiffener only resists loading in its stiffening direction, we see from equation 5 that

$$\sigma_x = E_x^s \left\{\frac{\partial u_o}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right\} \text{ for } -\left(\frac{t}{2}+h_s\right) \leq z \leq -\frac{t}{2} \tag{125}$$

where E_x^s is the modulus of the stiffener in its stiffening direction.

Combining equations 124 and 125 and performing the indicated integration results in

$$N_x^s = K_1 \frac{\partial u_o}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$K_1 = E_x^s \left(f_s t_{s_1} + t_s h_s + t_{s_1} d_s \right)$$

$$K_2 = E_x^s \left[f_s t_{s_1} \left(h_s + \frac{t}{2} - \frac{t_{s_1}}{2} \right) + \frac{t_s h_s}{2} \left(h_s + t \right) + \frac{d_s t_{s_1}}{2} \left(t_{s_1} + t \right) \right] \tag{126}$$

In a similar manner, we find that

$$M_x^s = \int_{\frac{-b_s}{2}}^{\frac{b_s}{2}} \int_{-\frac{t}{2} + h_s}^{-t/2} \sigma_x z dz dy = C_1 \frac{\partial u_o}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$C_1 = -E_x^s \left\{ f_s t_{s_1} \left(h_s + \frac{t}{2} - \frac{t_{s_1}}{2} \right) + \frac{h_s t_s}{2} \left(h_s + t \right) + \frac{d_s t_{s_1}}{2} \left(t + t_{s_1} \right) \right\} \tag{127}$$

$$C_2 = -E_x^s \left\{ t_{s_1} f_s \left[h_s \left(h_s + t - t_{s_1} \right) + t_{s_1} \left(\frac{t_{s_1}}{3} - \frac{t}{2} \right) + \frac{t^2}{4} \right] \right.$$

$$+ t_{s_1} d_s \left(\frac{t^2}{4} + \frac{t t_{s_1}}{2} + \frac{t_{s_1}^2}{3} \right)$$

$$\left. + h_s t_s \left(\frac{h_s^2}{3} + \frac{h_s t}{2} + \frac{t^2}{4} \right) \right\} \qquad (127)$$

HAT STIFFENERS

In the case of hat stiffeners, we consider the typical stiffener shown in figure 42. With the use of figure 42, we find that

$$N_x^s = \int_{-(f_s + d_s + t_s)}^{-(f_s + t_s)} \int_{-(t/2 + t_s)}^{-t/2} \sigma_x \, dz dy + \int_{(f_s + t_s)}^{-f_s} \int_{-(t/2 + h_s)}^{-t/2} \sigma_x \, dz dy$$

$$+ \int_{-f_s}^{f_s} \int_{-(t/2 + h_s)}^{-(t/2 + h_s - t_s)} \sigma_x \, dz dy + \int_{f_s}^{(f_s + t_s)} \int_{-(t/2 + h_s)}^{-t/2} \sigma_x \, dz dy$$

$$+ \int_{(f_s + t_s)}^{(f_s + t_s + d_s)} \int_{-(t/2 + h_s)}^{t/2} \sigma_x \, dz dy \qquad (128)$$

Substitution of equation 125 into 128 and performing the indicated integration results in

$$N_x^s = K_1 \frac{\partial u_o}{\partial x} + K_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$K_1 = 2E_x^s t_s \left(d_s + h_s + f_s\right)$$

$$K_2 = E_x^s t_s \left[d_s \left(t_s + t\right) + h_s \left(h_s + t\right) + 2 f_s \left(h_s + \frac{t}{2} - \frac{t_s}{2}\right)\right] \qquad (129)$$

Also

$$\int_{\frac{-b_s}{2}}^{\frac{b_s}{2}} \int_{-\left(t/2 + h_s\right)}^{-t_h} \sigma_x z \, dz dy = C_1 \frac{\partial u_o}{\partial x} + C_2 \frac{\partial^2 w}{\partial x^2}$$

where

$$C_1 = -t_s E_x^s \left[d_s \left(t_s + t \right) + h_s \left(h_s + t \right) + 2 f_s \left(\frac{t}{2} + h_s - \frac{t_s}{2} \right) \right]$$

$$C_2 = -2 t_s E_x^s \left\{ d_s \left(\frac{t_s^2}{3} + \frac{t t_s}{2} + \frac{t^2}{4} \right) + f_s \left[h_s \left(h_s + t - t_s \right) + t_s \left(\frac{t_s}{3} - \frac{t}{2} \right) + \frac{t^2}{4} \right] + h_s \left[\frac{t^2}{4} + \frac{t h_s}{2} + \frac{h_s^2}{3} \right] \right\} \qquad (130)$$

SECTION VI

MINIMUM WEIGHT ANALYSIS

GENERAL

A computer program, AC-12, was developed to calculate D_1 for hat- and Z-stiffened sheets, as well as the buckling parameter $N_x L^2$ of equation 115 for wide columns.

For the case of axial compression, the program equates the local buckling loads of the stiffened plate elements to insure that these elements fail simultaneously. Then, by equating this local panel strength to the panel strength in general instability, one may obtain a minimum weight concept for a given load requirement according to the simultaneous failure mode hypothesis for minimum weight structures.

The local buckling allowables are obtained from the following considerations:

1. For elements with three sides simply supported and one of the unloaded sides free, the buckling load can be obtained from

$$N_x = k_7 \, t_s^3 / f_s^2 \tag{131}$$

for Z-stiffeners with k_7 obtained by solving the transcendental equation presented in reference 6.

2. For elements with all four sides simply supported, we have from equation 37 that

$$N_x = k_f \, t_f^3 / b_s^2$$

$$N_x = k_s \, t_s^3 / h_s^2 \tag{132}$$

where

$$k_f = \frac{\pi^2 E_x^f}{6\left(1 - \nu_{xy}^f \nu_{xy}^f\right)}\left[\left(\frac{E_y^f}{E_x^f}\right)^{1/2} + 2\frac{G_{xy}^f}{E_x^f}\left(1 - \nu_{xy}^f \nu_{yx}^f\right) + \nu_{yx}^f\right]$$

$$k_s = \frac{\pi^2 E_x^s}{6\left(1 - \nu_{xy}^s \nu_{yx}^s\right)}\left[\left(\frac{E_y^s}{E_x^s}\right)^{1/2} + 2\frac{G_{xy}^s}{E_x^s}\left(1 - \nu_{xy}^s \nu_{yx}^s\right) + \nu_{yx}^s\right].$$

and the symbols s and f denote stiffener and face sheet, respectively.

ZEE STIFFENERS

For simultaneous buckling of the Z-stiffened elements, we require

$$k_7 \frac{t_s^2}{f_s^2} = k_s \frac{t_s^2}{h_s^2} \tag{133}$$

or

$$\left(f_s/h_s\right) = \left(k_7/k_s\right)^{1/2} = \beta_1$$

while for simultaneous buckling of the face and stiffener elements, we require

$$\left(t_s/h_s\right)\left(b_s/t_f\right) = \left(\frac{E_x^s}{E_x^f} \cdot \frac{k_f}{k_s}\right)^{1/2} = \alpha_1 \tag{134}$$

Equating the applied load in the stiffener to the allowable stiffener load yields

$$\frac{\bar{N}_x E_x^s t_s}{E_x^f t_f + E_x^s \frac{(2f_s + h_s)}{b_s} t_s} = \frac{k_s t_s^3}{h_s^2} \tag{135}$$

where $\bar{N}_x$ is the applied compressive force to the panel. With the use of these relationships, we find

$$h_s = b_s \left\{ \left(\frac{\bar{N}_x E_x^s b_s^2}{k_s \alpha_1^2 t_f^2} - E_x^f t_f \right) \Big/ \left[\left(2\beta_1 + 1 \right) \alpha_1 E_x^s t_f \right] \right\}^{1/2}$$

$$t_s = \alpha_1 h_s t_f / b_s \tag{136}$$

$$f_s = \beta_1 h_s$$

For a given applied force $\bar{N}_x$ and b_s, computer program AC-12 utilizes the relations of equation 136 to calculate $(N_x L^2)_{allowable}$, and the optimum configuration is

$$\frac{(N_x L^2)}{L^2} = \bar{N}_x$$

HAT STIFFENERS

In a similar manner, we find for hat stiffeners that $f_s \cong h_s/2$ and

$$h_s = b_s \left\{ \left(\frac{N_x E_x^s b_s^2}{k_s \alpha_1^2 t_f^2} - t_- E_x^f \right) \Big/ \left[\left(\beta_2 + 1.5 \right) \quad 2 \alpha_1 E_x^s t_f \quad \right] \right\}^{1/2}$$

$$f_s = h_s/2 \tag{137}$$

$$t_s = \alpha_1 t h_s/b_s$$

$$d_s = \beta_2 h_s$$

Using these relationships, program AC-12 calculates the desired properties for the hat-stiffened sheet in a manner identical to the Z-stiffened sheet.

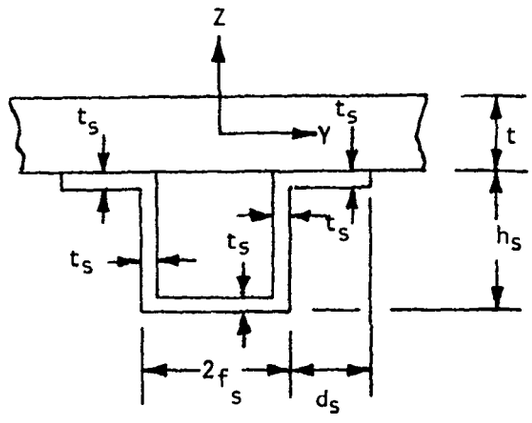


Figure 42. Typical Hat Stiffener

SECTION VII

THERMOELASTIC RELATIONSHIPS

In this analysis, it is assumed that the elastic and thermal response of each ply of an N-ply laminate is known. The quantities specifying the elastic and thermal response of a ply may be obtained from the data presented in volume I. The thermoelastic stress-strain relation for the i^{th} layer in a state of plane stress (stress components normal to the L-T plane are zero) is given by

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \begin{bmatrix} T_{11} & T_{12} & 0 \\ T_{12} & T_{22} & 0 \\ 0 & 0 & T_{33} \end{bmatrix}_i \begin{Bmatrix} \epsilon_L - \alpha_L \Delta T \\ \epsilon_T - \alpha_T \Delta T \\ \gamma_{LT} \end{Bmatrix}_i$$

where

$$\left.\begin{aligned} T_{11} &= E_L \Big/ \left(1 - \nu_{LT}\, \nu_{TL}\right) = E_L\, T_{22} \Big/ E_T \\ T_{12} &= \nu_{TL}\, E_L \Big/ \left(1 - \nu_{LT}\ \ _{TL}\right) = \nu_{LT}\, E_T \left(1 - \nu_{LT}\, \nu_{TL}\right) \\ T_{33} &= G_{LT} \\ \gamma_{LT} &= 2\, \epsilon_{LT} \end{aligned}\right\} \qquad (138)$$

Figure 43 shows the laminate in question.

Here, ν_{TL} denotes the magnitude of the strain in the longitudinal direction due to a unit strain in the transverse direction; ν_{LT} denotes the magnitude of the strain in the transverse direction due to a unit strain in the longitudinal direction.

To obtain the stress-strain relations for a lamina for the case where the applied stress makes an angle θ_i to the natural lamina axis, equation 138 must be subjected to a coordinate transformation. The case under consideration is illustrated in figure 44.

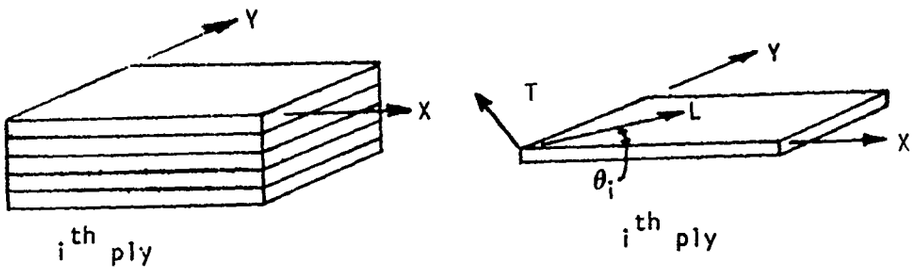


Figure 43. Coordinates of i^{th} Ply in Natural Lamina and Laminate Systems

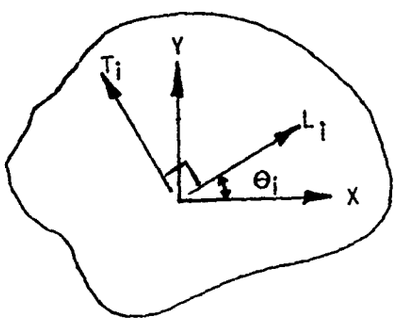


Figure 44. Relationship Between Laminae and Laminate Coordinate Systems

The stress-strain relations for the typical i^{th} ply of the laminate shown in figure 43 is assumed to be of the form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = \left[B\right]_i \begin{Bmatrix} \epsilon_x - \alpha_x \Delta T \\ \epsilon_y - \alpha_y \Delta T \\ \epsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_i \tag{139}$$

or $$\left\{\sigma\right\}_i = \left[B\right]_i \left\{\epsilon\right\}_i - \left[B\right]_i \left\{\alpha\right\}_i \Delta T$$

where the matrix $[B]_i$ of order 3 x 3 is yet to be determined. It is well known that the ply stress and strain components transform from one coordinate system (X, Y, Z) to another coordinate system (L, T, Z) according to the transformation law of a tensor of order two (reference 1). Hence,

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \left[D\right]_i \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i , \quad \begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \epsilon_{LT} \end{Bmatrix}_i = \left[D\right]_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix}_i ,$$

$$\begin{Bmatrix} \alpha_L \\ \alpha_T \\ \frac{\alpha_{LT}}{2} \end{Bmatrix}_i = \left[D\right]_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{\alpha_{xy}}{2} \end{Bmatrix} \tag{140}$$

where

$$\left[D\right]_i = \begin{bmatrix} m_i^2 & n_i^2 & 2\,m_i n_i \\ n_i^2 & m_i^2 & -2\,m_i n_i \\ -m_i n_i & m_i n_i & m_i^2 - n_i^2 \end{bmatrix} \tag{141}$$

and $\quad n_i = \sin\theta_i, \quad m_i = \cos\theta_i$

Substituting equation 140 into 138 and performing the appropriate matrix manipulations, we find that $[B]_i$ of equation 139 is

$$\left[B\right]_i = \left[D\right]_i^{-1} \left[T\right]_i \left[D\right] \tag{142}$$

By performing the indicated multiplication for equation 142, it is found that the elements of the $[B]_i$ matrix are

$$\begin{aligned}
B_{11} &= T_{11}m^4 + 2\,(T_{12} + 2\,T_{33})\,m^2n^2 + T_{22}n^4 \\
B_{12} &= (T_{11} + T_{22} - 4T_{33})m^2n^2 + T_{12}\,(m^4 + n^4) = B_{21} \\
B_{13} &= (T_{11} - T_{12} - 2T_{33})nm^3 + (T_{12} - T_{22} + 2\,T_{33})\,n^3m = B_{31} \\
B_{22} &= T_{11}\,n^4 + 2\,(T_{12} + 2\,T_{33})\,n^2m^2 + T_{22}m^4 \\
B_{23} &= (T_{11} - T_{12} - 2T_{33})\,n^3m + (T_{12} - T_{22} + 2\,T_{33})\,nm^3 = B_{32} \\
B_{33} &= (T_{11} + T_{22} - 2T_{12} - 2T_{33})\,n^2m^2 + T_{33}\,(m^4 + n^4)
\end{aligned} \tag{143}$$

We will now consider a laminate consisting of n plies with natural axes making various angles θ_i, with respect to the reference (X, Y) axes of the laminate. We impose the restriction that the plies are arranged symmetrically with respect to the center plane of the laminate. Because of this restriction, there is no coupling between in-plane loading and out-of-plane deformations.

The laminate plate shown in figure 43 is assumed to be subjected to applied stresses (ir plane) and temperature changes (uniform) which do not result in curvature of the plies. For a laminate of thickness t, the stress resultants are defined by

$$N_x = \int_{-t/2}^{t/2} \sigma_x \, dz, \qquad N_y = \int_{-t/2}^{t/2} \sigma_y \, dz, \qquad N_{xy} = \int_{-t/2}^{t/2} \sigma_{xy} \, dz \tag{144}$$

Integrating equation 139 across the laminate thickness and assuming the stress components to be constant across each lamina and the strain components not to vary with z, we obtain

$$\{N\} = [\bar{B}] \{\epsilon\} - \Delta T \{C\} \tag{145}$$

where

$$N_x = \sum_{i=1}^{n} \sigma_{x_i} h_i, \qquad N_y = \sum_{i=1}^{n} \sigma_{y_i} h_i, \qquad \sum_{i=1}^{n} \sigma_{xy_i} h_i$$

$$\bar{B}_{jk} = \sum_{i=1}^{n} \left(B_{jk}\right)_i h_i \tag{146}$$

$$C_x = \sum_{i=1}^{n} \left(\alpha_x B_{11} + \alpha_y B_{12} + \alpha_{xy} B_{13}\right)_i h_i$$

$$C_y = \sum_{i=1}^{n} \left(\alpha_x B_{21} + \alpha_y B_{22} + \alpha_{xy} B_{23} \right)_i h_i$$

$$C_{xy} = \sum_{i=1}^{n} \left(\alpha_x B_{31} + \alpha_y B_{32} + \alpha_{xy} B_{33} \right)_i h_i \qquad (146)$$

and h_i denotes the thickness of the ith ply.

The coefficients of thermal expansion α_x^c , α_y^c , α_{xy}^c for the n-ply laminate are obtained from equation 145 by setting the applied in-plane resultants equal to zero. Hence,

$$\begin{Bmatrix} \alpha_x^c \\ \alpha_y^c \\ \alpha_{xy}^c \end{Bmatrix} = \left\{\alpha^c\right\} = \frac{\left\{\epsilon\right\}}{\Delta T} = \left[\overline{B}\right]^{-1} \left\{C\right\} \qquad (147)$$

Then the general stress-strain relation for an n-layered laminate subjected to extensional deformation takes on the form

$$\left\{N\right\} = \left[\overline{B}\right]\left\{\epsilon\right\} - \left[\overline{B}\right]\left\{\alpha\right\}\Delta T = \left[\overline{B}\right]\left\{\epsilon - \alpha^c \Delta T\right\} \qquad (148)$$

Therefore, for any combination of applied in-plane loadings and uniform temperature changes, the corresponding strain field for the laminate is obtained from equation 148.

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} = \left\{\epsilon\right\} = \left[\overline{B}\right]^{-1} \left\{N\right\} + \Delta T \left\{\alpha^c\right\} = \left\{\epsilon\right\}_i, \text{ for } i = 1, 2, \text{---}N \qquad (149)$$

To calculate the stresses in the i^{th} ply due to the strain field of equation 149 for all plies, equation 138 is employed; hence,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_i = \left[B\right]_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} - \left[B\right]_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \Delta T \tag{150}$$

These stresses are in the direction of the laminate reference axes X, Y. The stresses in each ply with respect to the natural axes of the ply are obtained from

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \left[T\right]_i \left[D\right]_i \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} - \left[T\right]_i \left[D\right]_i \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \Delta T \tag{151}$$

For the case when only a uniform temperature change is present, we find from equations 149 and 151 that the i^{th} ply stresses become

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \sigma_{LT} \end{Bmatrix}_i = \left[T\right]_i \left[D\right]_i \left[\begin{Bmatrix} \alpha_x^c \\ \alpha_y^c \\ \alpha_{xy}^c \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_i \right] \Delta T \tag{152}$$

or the ply stresses in its natural coordinate system are proportional to the differences between the laminate coefficients of thermal expansion, as defined by equation 147, and the ply coefficients of thermal expansion.

A computer program, AC-40, has been written to perform these calculations for $\{\alpha^c\}$ for any crossplied laminates. The program listing and sample data and output sheets are presented in another section of volume III. Correlations of this theory with available test data and typical design curves are described in volume I of this report.

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SECTION VIII
JOINT ANALYSIS

SINGLE-LAP ADHESIVE JOINTS

The derivation of the single-lap shear joint stress distribution equation for relatively flexible adhesive layers is shown in reference 8 and will not be repeated here.

The following assumptions were used:

1. The laminate and adhesive are isotropic and elastic.
2. The laminate deformation is cylindrical.
3. The adhesive is relatively flexible.
4. The transverse normal strain and shear strains in the laminate are negligible in comparison with those of the adhesive.

The results are

$$\bar{f}_a^s = \frac{N_x}{4L_a}\left[\beta(1+3\alpha)\frac{\cosh\frac{2\beta x}{L_a}}{\sinh\beta} + 3(1-\alpha)\right]$$

where

$$\beta = \frac{L_a}{t}\left[\frac{2(1-\nu_{xy}^2)G_a t}{E_x t_a}\right]^{1/2} \qquad (153)$$

$$\alpha = \frac{1}{1 + 2\sqrt{2}\tanh\left[\frac{L_a}{2}\sqrt{\frac{3(1-\nu_{xy}^2)N_x}{2E_x t}}\right]}$$

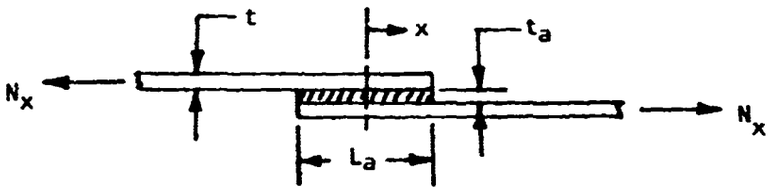


Figure 45. Typical Single Lap Joint

The axial stress distribution is also given in the reference but will not be presented here since it was not used in our analysis. The maximum shear stress occurs at $x = \pm L_a/2$:

$$\bar{f}_a^s = \frac{1}{4}\left[\beta(1+3\alpha)\coth\beta + 3(1-\alpha)\right]\frac{N_x}{L_a} \qquad (154)$$

CONTINUOUS TAPER SCARF JOINT

The derivation of the continuous taper scarf joint stress distribution is presented. The following assumptions are made:

1. The adhesive is isotropic and elastic.
2. The adhesive thickness is small.
3. The joint is everywhere in a plane state of stress.
4. The stress state is not a function of y.

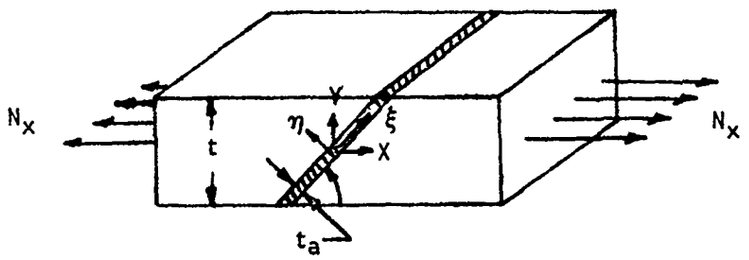


Figure 46. Typical Scarf Joint

From assumption 2 we see that for compatibility the strains at the bondline in the ξ direction for both the laminate and adhesive must be equal.

$$\epsilon_\xi^a = \epsilon_\xi \tag{155}$$

From assumption 4 we see that

$$\sigma_\eta^a = \sigma_\eta = \sigma_x \sin^2\phi$$

Also $$\tau_{\xi\eta} = \sigma_x \sin\phi\cos\phi$$

$$\sigma_\xi = \sigma_x \cos^2\phi$$

The constitutive equations yield

$$\epsilon_\xi^a = \frac{1}{E_a}\left(\sigma_\xi^a - \nu_a \sigma_\eta^a\right) = \frac{1}{E_a}\left(\sigma_\xi^a - \nu_a \sigma_x \sin^2\phi\right) \tag{156a}$$

$$\epsilon_\xi = \frac{1}{E_x}\left(\sigma_\xi - \nu_{xy}\sigma_\eta\right) = \frac{\sigma_x}{E_x}\left(\cos^2\phi - \nu_{xy}\sin^2\phi\right) \tag{156b}$$

Substituting equations 156a and 156b into equation 155 yields

$$\sigma_{\xi}^{a} = \sigma_x \left[\nu_a \sin^2\phi + \frac{E_a}{E_x}\left(\cos^2\phi - \nu_{xy} \sin^2\phi\right)\right]$$

The shearing stress is

$$\tau_{\xi\eta} = \sigma_x \sin\phi \cos\phi$$

Using Tresca's maximum shear criteria, we find that

$$\bar{f}^{s}_{a,max} = \left[\left(\frac{\sigma_{\xi}^{a} - \sigma_{\eta}^{a}}{2}\right)^2 + \left(\tau_{\xi\eta}^{a}\right)^2\right]^{1/2}$$

$$= \frac{N_x}{t}\left\{\frac{1}{4}\left[\sin^2\phi\left(1 - \nu_a + \nu_{xy}\frac{E_a}{E_x}\right) - \frac{E_a}{E_x}\cos^2\phi\right]^2 + \sin^2\phi\cos^2\phi\right\}^{\frac{1}{2}} \quad (157)$$

DOUBLE-LAP ADHESIVE JOINT

The derivation of the double-lap shear joint stress distribution is presented. The assumptions are:

1. The adherend and adhesive are isotropic and elastic.
2. The adherend displacements, u_1 and u_2, are small and a function of x only.
3. Only shearing deformation in the adhesive is considered.

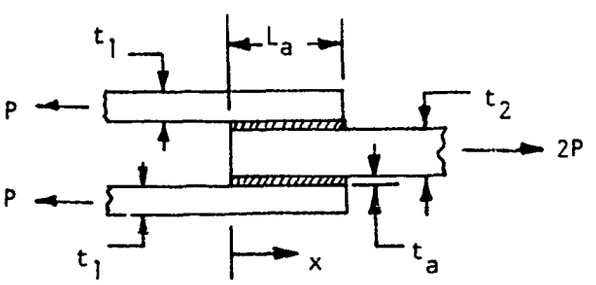


Figure 47. Typical Double Lap Joint

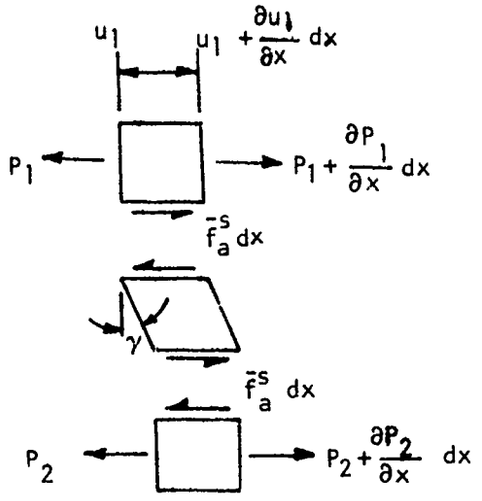


Figure 48. Typical Load Transfer Through Adhesive

Considering assumptions 2 and 3,

$$\gamma = \frac{u_2 - u_1}{t_a} \tag{158}$$

and the constitutive equation

$$\bar{f}_a^s = G_a \gamma = \frac{G_a}{t_a} \left(u_2 - u_1 \right) \tag{159}$$

where

G_a = shear modulus of the adhesive.

Equilibrium for the upper element requires

$$\bar{f}_a^s dx + \frac{\partial P_1}{\partial x} dx = 0 = \left[\frac{G_a}{t_a} \left(u_2 - u_1 \right) + \frac{\partial P_1}{\partial x} \right] dx \tag{160}$$

The incremental elongation of the adherends can be expressed as

$$\left(\frac{\partial P_1}{\partial x}\,dx\right)\frac{dx}{t_1E_1} = \frac{\partial u_1}{\partial x}\,dx \tag{161a}$$

$$2\left(\frac{\partial P_2}{\partial x}\,dx\right)\frac{dx}{t_2E_2} = \frac{\partial u_2}{\partial x}\,dx \tag{161b}$$

where E_1 and E_2 are the Young's moduli of extension of the adherends. Noting assumption 2, we see that total derivatives can be used. Differentiating equation 160 and using equations 161a and 161b, we obtain

$$\frac{d^2P_1}{dx^2} + \frac{G_a}{t_a}\left[\frac{2\,\frac{dP_2}{dx}}{t_2E_2} - \frac{\frac{dP_1}{dx}}{t_1E_1}\right] = 0$$

Equilibrium for the joint requires that the condition $P = P_1 + P_2$ be satisfied; hence,

$$\frac{d^2P_1}{dx^2} - \frac{G_a}{t_a}\left[\frac{1}{t_1E_1} + \frac{2}{t_2E_2}\right]P_1 = -\frac{2G_a}{t_at_2E_2}P \tag{162}$$

The procedure followed is that of reference 9. However, there is a difference in the signs of the last two terms. The solution of this differential equation is

$$P_1 = A\sinh\omega x + B\cosh\omega x + \beta P \tag{163}$$

$$\omega = \sqrt{\frac{G_a}{t_a}\left(\frac{1}{t_1E_1} + \frac{2}{t_2E_2}\right)}$$

$$\frac{1}{\beta} = \left(1 + \frac{t_2E_2}{2t_1E_1}\right)$$

A, B = undetermined coefficients.

The boundary conditions are

$$P_1(0) = P \tag{164a}$$

$$P_1(L_a) = 0 \tag{164b}$$

Solving equation 163 for the undetermined coefficients yields

$$P_1 = \left[-\frac{(1-\beta)\cosh \omega L_a + \beta}{\sinh \omega L_a} \sinh \omega x + (1-\beta)\sinh \omega x + \beta \right] P$$

From equation (160)

$$\bar{f}_a^s = -\frac{dP_1}{dx} = \left[\frac{(1-\beta)\cosh \omega L_a + \beta}{\sinh \omega L_a} \cosh \omega x - (1-\beta)\sinh \omega x \right] \omega P \tag{165}$$

Practical considerations show that

$$\omega L_a > 5$$

$$0 < \beta < 1$$

Thus we see that

$$(1-\beta)\cosh L_a >> \beta$$

$$\cosh \omega L_a \cong \sinh \omega L_a$$

and equation (165) becomes

$$\bar{f}_a^s \cong (1-\beta) \left[\frac{\cosh \omega L_a}{\sinh \omega L_a} \cosh \omega x - \sinh \omega x \right] \omega P$$

This equation attains a maximum at either $x = 0$ or $x = L_a$, depending on β. Evaluating at $x = 0$, we get

$$\bar{f}_a^s(0) = (1-\beta)\omega P$$

Evaluating equation (165) at $x = L_a$

$$\bar{f}_a^s (L_a) = \left[(1-\beta) \left(\frac{\cosh^2 \omega L_a - \sinh^2 \omega L_a}{\sinh \omega L_a} \right) + \beta \coth \omega L_a \right] \omega P$$

$$= \left[(1-\beta) \frac{1}{\sinh \omega L_a} + \beta \coth \omega L_a \right] \omega P$$

Therefore,

$$\bar{f}_a^s (L_a) \cong \beta \omega P$$

Letting $P = N_x/2$, the equations become

$$\bar{f}_a^s(0) = (1 - \beta)\, \omega L_a \cdot \frac{N_x}{2L_a} = (1 - \beta)\, \omega N_x/2 \qquad (166a)$$

$$\bar{f}_a^s(L_a) = \beta \omega L_a \frac{N_x}{2L_a} = \beta \omega N_x/2 \qquad (166b)$$

The maximum shear stress becomes

$$\bar{f}_{a,\,max}^s = \beta' \omega L_a \frac{N_x}{2L_a} = \beta' \omega N_x/2 \qquad (167)$$

where β' is the larger of β or $1-\beta$

DOUBLE-LAP VARIABLE STIFFNESS ADHESIVE JOINT

The derivation of the double-lap two-adhesive joint, as shown by figure 49, is presented in the following paragraphs.

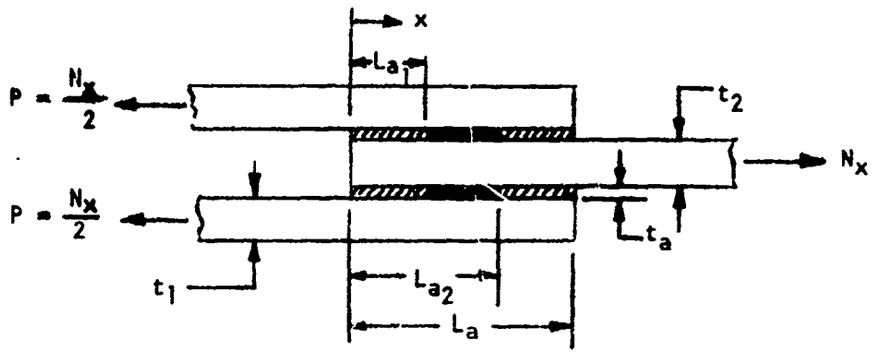


Figure 49. Typical Variable Stiffness Adhesive Double Lap Joint

The procedure used here will essentially follow Szepe's (reference 9). However, his equations were modified in reference 10 and developed in the previous section; thus, from equation 162 and figure 58 we have

$$P_{1,xx} - \frac{G_a}{t_a}\left(\frac{1}{t_1E_1} + \frac{2}{t_2E_2}\right) P_1 = -2\frac{G_a}{t_a t_2 E_2} P \qquad (168)$$

The solution to this equation is

$$P_1 = A \sinh \omega x + B \cosh \omega x + \beta P \qquad (169)$$

where

$$\omega^2 = \frac{G_a}{t_a}\left(\frac{1}{t_1E_1} + \frac{2}{t_2E_2}\right)$$

$$\beta = \frac{2\, t_1E_1}{2\, t_1E_1 + t_2E_2}$$

Considering the three regions designated as one, two, and three, respectively, as

$$0 \leq x \leq L_{a1}; \quad L_{a1} \leq x \leq L_{a2}; \quad L_{a2} \leq x \leq L_a$$

we get

$$\begin{aligned} P_{11} &= A_1 \sinh \omega_1 x + B_1 \cosh \omega_1 x + \beta P \qquad 0 \leq x \leq L_{a1} \\ P_{12} &= A_2 \sinh \omega_2 x + B_2 \cosh \omega_2 x + \beta P \qquad L_{a_1} \leq x \leq L_{a2} \\ P_{13} &= A_3 \sinh \omega_1 x + B_3 \cosh \omega_1 x + \beta P \qquad L_{a2} \leq x \leq L_a \end{aligned} \tag{170}$$

noting that the shearing stress in the adhesive is given by

$$\begin{aligned} \bar{f}_a^s &= P_{1,\,x} \\ \bar{f}_a^{s1} &= \omega_1 A_1 \cosh \omega_1 x + \omega_1 B_1 \sinh \omega_1 x \\ \bar{f}_a^{s2} &= \omega_2 A_2 \cosh \omega_2 x + \omega_2 B_2 \sinh \omega_2 x \\ \bar{f}_a^{s3} &= \omega_1 A_3 \cosh \omega_1 x + \omega_1 B_3 \sinh \omega_1 x \end{aligned} \tag{171}$$

The unknown coefficients A_1, A_2, A_3, B_1, B_2, and B_3 can be obtained from the known boundary conditions,

$$\begin{aligned} P_{11}(0) &= P \\ P_{11}\left(L_{a1}\right) &= P_{12}\left(L_{a1}\right) \end{aligned} \tag{172}$$

$$\frac{f_a^{s1}\left(L_{a1}\right)}{G_{a1}} = \frac{f_a^{s2}\left(L_{a1}\right)}{G_{a2}}$$

$$P_{12}\left(L_{a2}\right) = P_{13}\left(L_{a2}\right)$$

$$\frac{f_a^{s2}\left(L_{a2}\right)}{G_{a2}} = \frac{f_a^{s3}\left(L_{a2}\right)}{G_{a1}}$$

$$P_{13}\left(L_a\right) = 0$$

Substitution yields the following equations

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ S_{11} & C_{11} & -S_{21} & -C_{21} & 0 & 0 \\ C_{11} & S_{11} & -\frac{\omega_1}{\omega_2}C_{21} & -\frac{\omega_1}{\omega_2}S_{21} & 0 & 0 \\ 0 & 0 & S_{22} & C_{22} & -S_{12} & -C_{12} \\ 0 & 0 & C_{22} & S_{22} & -\frac{\omega_2}{\omega_1}C_{12} & -\frac{\omega_2}{\omega_1}S_{12} \\ 0 & 0 & 0 & 0 & S_{13} & C_{13} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{Bmatrix} = P \begin{Bmatrix} 1-\beta \\ 0 \\ 0 \\ 0 \\ 0 \\ -\beta \end{Bmatrix} \qquad (173)$$

The solutions of these equations are

$$A_1 = \begin{Bmatrix} -S_{11}\,S_{21} + \dfrac{\omega_1}{\omega_2} C_{11}\,C_{21} \\ -S_{11}\,C_{21} + \dfrac{\omega_1}{\omega_2} C_{11} S_{21} \\ 0 \end{Bmatrix}^T \begin{Bmatrix} A_2 \\ B_2 \\ A_3 \end{Bmatrix} \tag{174}$$

$$B_1 = (1 - \beta)\ P \tag{175}$$

$$\begin{Bmatrix} A_2 \\ B_2 \\ A_3 \end{Bmatrix} = -\frac{P}{\lambda} \left[\begin{array}{c|c|c} \lambda_{22}\lambda_{33} - \dfrac{\omega_1}{\omega_2}\lambda_{21}\lambda_{23} & -\lambda_{12}\lambda_{33} & \lambda_{12}\lambda_{23} \\ \hline \dfrac{\omega_1}{\omega_2}\lambda_{22}\lambda_{23} - \lambda_{21}\lambda_{33} & \lambda_{11}\lambda_{33} & -\lambda_{11}\lambda_{23} \\ \hline \dfrac{\omega_1}{\omega_2}\left(\lambda_{21}^2 - \lambda_{22}^2\right) & \dfrac{\omega_1}{\omega_2}\left(\lambda_{12}\lambda_{22} - \lambda_{11}\lambda_{21}\right) & \lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12} \end{array}\right] \begin{Bmatrix} 1-\beta \\ \beta C_{12} \\ \beta S_{12} \end{Bmatrix} \tag{176}$$

$$B_3 = -\frac{S_{13}}{C_{13}} A_3 - \frac{\beta}{C_{13}} P \tag{177}$$

where

$$S_{11} = \sinh \omega_1 L_{a_1} \qquad C_{11} = \cosh \omega_1 L_{a_1}$$

$$S_{12} = \sinh \omega_1 L_{a_2} \qquad C_{12} = \cosh \omega_1 L_{a_2}$$

$$S_{21} = \sinh \omega_2 L_{a_1} \qquad C_{21} = \cosh \omega_2 L_{a_1}$$

$$S_{22} = \sinh \omega_2 L_{a_1} \qquad C_{22} = \cosh \omega_2 L_{a_2}$$

$$S_{13} = \sinh \omega_1 L_a \qquad C_{13} = \cosh \omega_1 L_a$$

$$\lambda_{11} = - C_{11} S_{21} + \frac{\omega_1}{\omega_2} S_{11} C_{21} \qquad \lambda_{12} = - C_{11} C_{21} + \frac{\omega_1}{\omega_2} S_{11} S_{21}$$

$$\lambda_{21} = S_{22} C_{13} \qquad \lambda_{22} = C_{22} C_{13}$$

$$\lambda_{23} = C_{12} S_{13} - S_{12} C_{13} \qquad \lambda_{33} = S_{12} S_{13} - C_{12} C_{13}$$

$$\lambda = \lambda_{11} \left(\lambda_{22} \lambda_{33} - \frac{\omega 1}{\omega_2} \lambda_{21} \lambda_{23} \right) - \lambda_{12} \left(\lambda_{21} \lambda_{33} - \frac{\omega_1}{\omega_2} \lambda_{22} \lambda_{23} \right)$$

The loading of the laminate, $P_1(x)$ and $P_2(x)$, where $P_1(x) + P_2(x) = 2P$, and the adhesive $\bar{f}_a^s(x)$ can now be found. To check these equations, we examine the case where a single adhesive is considered, i.e.,

$$P_{11}(x) \quad \text{for } 0 \leq x \leq L_{a1} \quad ; \qquad P_{13}(x) \quad \text{for } L_{a1} \leq x \leq L_a$$

$$L_{a2} = L_{a1} = L_a/2$$

For this case, the equations reduce to those of reference 3. We examine next the equations for

$$P_{12}(x) \text{ for } 0 \le x \le L_a \text{ and } L_{a1} = 0 \text{ ; with } L_{a2} = L_a$$

For this case, the equations also reduce to those of reference 3.

To avoid computational difficulties which occur, the equations can be written in the following form. Let $\tilde{\omega} = \dfrac{\omega_1}{\omega_2}$.

$$\begin{bmatrix} S_{11} & C_{11} & -S_{21} & -C_{21} & 0 & 0 \\ -C_{11} & -S_{11} & \tilde{\omega}C_{21} & \tilde{\omega}S_{21} & 0 & 0 \\ 0 & 0 & S_{22} & C_{22} & -S_{12} & -C_{12} \\ 0 & 0 & -C_{22} & -S_{22} & C_{12}/\tilde{\omega} & S_{12}/\tilde{\omega} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{13} & C_{13} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1-\beta \\ -\beta \end{Bmatrix} \qquad (178)$$

Noting the following relationships,

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh (x \pm y) = \sinh x \cdot \cosh y \pm \cosh x \cdot \sinh y$$

$$\cosh (x \pm y) = \cosh x \cdot \cosh y \pm \sinh x \cdot \sinh y$$

we obtain the following:

$$\begin{Bmatrix} A_1 \\ B_1 \end{Bmatrix} = \begin{bmatrix} C_{41} - (1-\tilde{\omega})C_{11}C_{21} & S_{41} - (1-\tilde{\omega})C_{11}S_{21} \\ S_{41} + (1-\tilde{\omega})S_{11}C_{21} & C_{41} + (1-\tilde{\omega})S_{11}S_{21} \end{bmatrix} \begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} \tag{179a}$$

$$\begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} = \frac{1}{\tilde{\omega}} \begin{bmatrix} C_{42} + (1-\tilde{\omega})S_{12}S_{22} & -S_{42} + (1-\tilde{\omega})C_{12}S_{22} \\ -S_{42} + (1-\tilde{\omega})S_{12}S_{22} & C_{42} - (1-\tilde{\omega})C_{12}C_{22} \end{bmatrix} \begin{Bmatrix} A_3 \\ B_3 \end{Bmatrix} \tag{179b}$$

$$\left[\begin{array}{c|c} -S_{44} + (1-\tilde{\omega})(S_{11}C_{55} - S_{12}C_{66}) + (1-\tilde{\omega})^2 S_{11}S_{12}S_{24} & \\ \hline S_{13} & \end{array}\right.$$

$$\left.\begin{array}{c|c} & C_{44} + (1-\tilde{\omega})(S_{11}S_{55} - C_{12}C_{66}) + (1-\tilde{\omega})^2 S_{11}C_{12}S_{24} \\ \hline & C_{13} \end{array}\right] \begin{Bmatrix} A_3 \\ B_3 \end{Bmatrix} = P \begin{Bmatrix} \tilde{\omega}(1-\beta) \\ -\beta \end{Bmatrix} \tag{179c}$$

where

$C_{41} = \cosh[(\omega_2-\omega_1)C_1]$ | $S_{41} = \sinh[(\omega_2-\omega_1)C_1]$

$C_{42} = \cosh[(\omega_2-\omega_1)C_2]$ | $S_{42} = \sinh[(\omega_2-\omega_1)C_2]$

$C_{44} = \cosh[(\omega_2-\omega_1)(C_2-C_1)]$ | $S_{44} = \sinh[(\omega_2-\omega_1)(C_2-C_1)]$

$C_{55} = \cosh[\omega_2C_1 - (\omega_2-\omega_1)C_2]$ | $S_{24} = \sinh[\omega_2(C_2-C_1)]$

$C_{66} = \cosh[\omega_2C_2 - (\omega_2-\omega_1)C_1]$ | $S_{55} = \sinh[\omega_2C_1 - (\omega_2-\omega_1)C_2]$

It should be noted that the equation leading to the solution of A_3, B_3 is still almost singular, and double-precision arithmetic must be used on a computer.

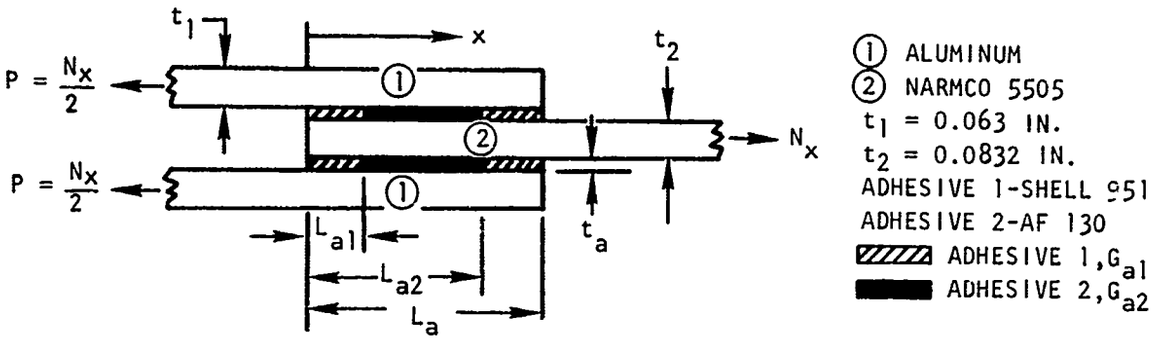
These equations can now be used to determine the loading in the laminate and the shear stress in the adhesive. Figure 50 shows a typical distribution of $\bar{f}_a^s$. The discontinuities of the shearing stress at $x/L_a = 0.25, 0.75$ are violations of equilibrium principles. However, considering the following:

1. only six free constants are available to satisfy boundary conditions,

2. finite element solution of a similar single-lap variable joint using an energy approach (reference 12) also shows an abrupt change in stress,

3. the thinness of the adhesive with respect to the joint members,

it is obvious that this solution is basically correct although the actual stresses would be more like the dotted line shown.



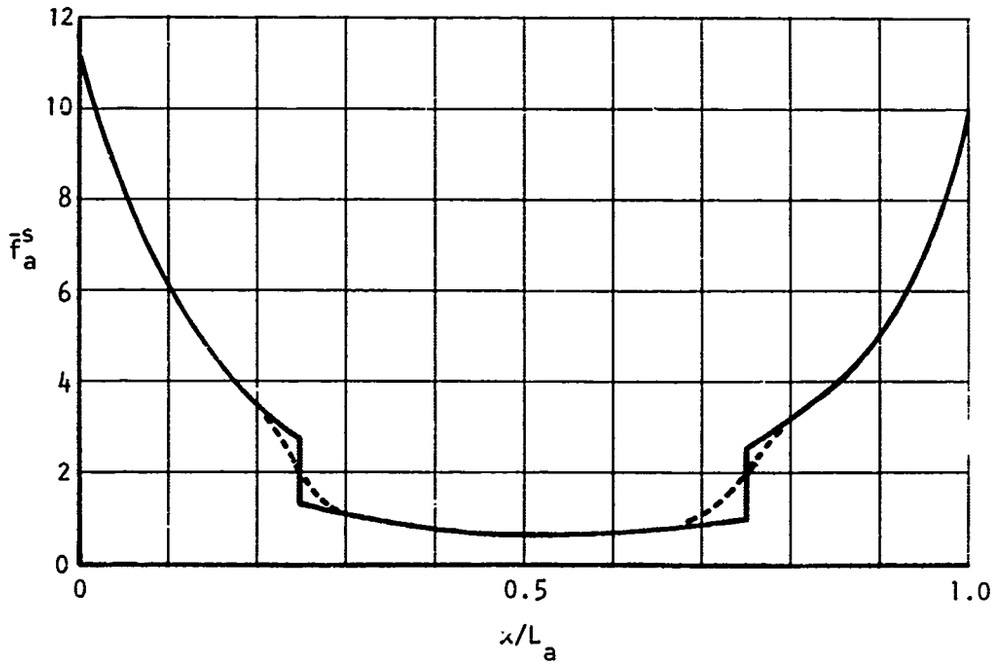


Figure 50. Double-Lap Variable Adhesive - Typical Distribution of $\bar{f}_a^s$ versus x/L_a

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APPENDIX

COMPUTER PROGRAMS

FOREWORD

This appendix contains a number of computer programs that were generated during the theoretical development efforts of this contract. They are quite general in applicability and are presented here separately from the main body of this volume to facilitate their usage. For each program, there are presented an abstract, the input data format, the output data format, and an example problem. The following is a list of the program numbers and titles.

No.	Title
AC-2	Extensional and Flexural Constants for Balanced Symmetrical Laminates of the Form $[0,90/\pm\theta/0,90]_{nS}$
AC-5	H/C Sandwich Panel Stability Under In-Plane Biaxial Loading
AC-7	Analysis of Simply Supported Orthotropic Sandwich Flat Panels Under Pressure or Pressure Plus In-Plane Loads
AC-11	H/C Sandwich Panel Stability for In-Plane Shear Loading
AC-12	Hat- and Z-Stiffened Panel Stability and Stiffness
AC-40	N-Ply Laminate Thermoelastic Analysis

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TABLE OF CONTENTS

Section		Page
	PROGRAM AC-2 - EXTENSIONAL AND FLEXURAL CONSTANTS FOR BALANCED SYMMETRICAL LAMINATES OF THE FORM $[0,90/\pm\theta/0,90]_{nS}$	
	Abstract	121
	Program Input Data	122
	Example Problem	130
	PROGRAM AC-5 - H/C SANDWICH PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING	
	Abstract	135
	Program Input Data	135
	Output	137
	Example Problem	142
	AC-7 - ANALYSIS OF SIMPLY SUPPORTED ORTHOTROPIC SANDWICH FLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS	
	Abstract	151
	Program Input Data	151
	Output	154
	Example Problem	160
	AC-11 - H/C SANDWICH PANEL STABILITY FOR IN-PLANE LOADING	
	Abstract	181
	Program Input Data	181
	Output	183
	Example Problem	190
	AC-12 - HAT- and Z-STIFFENED PANEL STABILITY AND STIFFNESS	
	Abstract	195
	Program Input Data	195
	Example Problem	202

Section | Page

AC-40 - N-PLY LAMINATE THERMOELASTIC ANALYSIS

Abstract 209
Program Input Data 209
Output 209
Example Problem 215

COMPUTER PROGRAM AC-2

EXTENSIONAL AND FLEXURAL CONSTANTS FOR
BALANCED SYMMETRICAL LAMINATES OF THE FORM

$$\left[0,90/\pm\theta/0,90\right]_{nS}$$

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES COMPUTER PROGRAM DECK NO. AC-2

ABSTRACT

Fortran program deck No. AC-2 is a program to calculate the laminate flexural and extensional constants; the honeycomb (H/C) sandwich parameters α and ψ; and the magnitudes of the flexural rigidity elements D_{16} and D_{26} relative to the D_{11} and D_{22} rigidities by using, as input data, the average mechanical property values for an individual unidirectional lamina and the geometric properties of the total laminate.

The program is somewhat restricted as to the type of laminate configuration that can be analyzed. The restrictions are:

1. The laminate layup is symmetric relative to its midplane.
2. The layups must be balanced with respect to $+\theta$ and $-\theta$ plies (i.e., there must be a $+\theta$ ply for every $-\theta$ ply).
3. The ply orientations only include one angle θ other than 0 and 90 degrees.
4. The laminate is made up of an even number of identical sets of ply orientations.
5. Every ply is assumed to have the same nominal ply thickness.
6. Every ply is assumed to have the same average mechanical properties.

The method used to find the "exact" flexural constants differs from the current state-of-the-art method in that no summations are involved. The summations are replaced by algebraic total sums. This means that, for a laminate with many plies, the computing time will be shortened considerably. Also, the computer results for a laminate with many plies will be more accurate because of the reduction of roundoff errors.

The "exact" flexural constants mentioned come from the standard state-of-the-art method. The program, however, also solves for the laminate flexural constants that result from the laminate extensional constants. The latter flexural constants are only approximations and consider the rigidities D_{16} and D_{22} to be zero. However, if a large number of plies is considered (i.e., a large number of sets), the "exact" and extensional constant methods will yield virtually the same results.

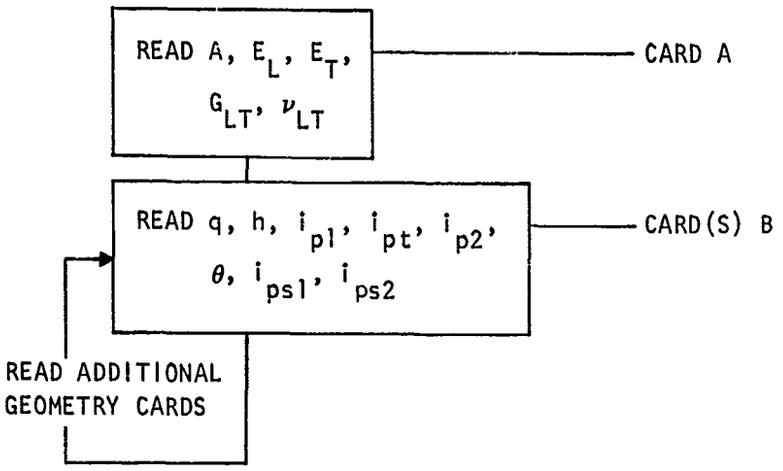
PROGRAM INPUT DATA

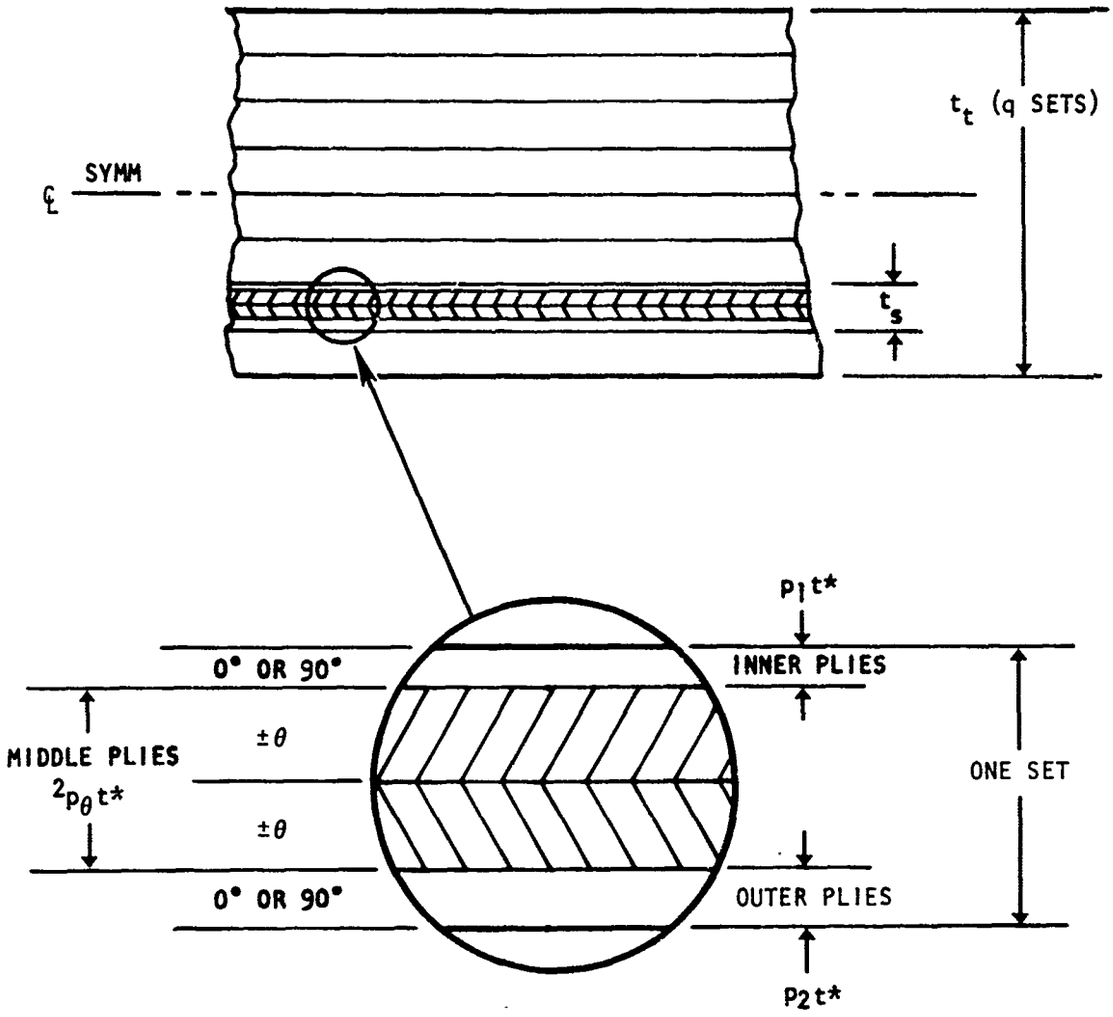
The input data are in two phases. The first phase (data card A) consists of giving the computer the average mechanical properties of a single unidirectional lamina. These properties are E_L (longitudinal Young's modulus), E_T (transverse Young's modulus), G_{LT} (shear modulus), and ν_{LT} (Poisson's ratio). An indicator (A) is also part of the data on the first card. Its purpose is to tell the computer which kind of average mechanical property data are being used, (i.e., A = 1.0 for average compression data, A = 0.0 for average tension-compression data, and A = 1.0 for average tension data). The correct order of the data on the first data card is A, E_L, E_T, G_{LT} and ν_{LT}. The input format is (5F12.0). An example phase I green sheet is shown following the text in this section. Note that all five input values need decimal points.

The second phase of input data (remaining data cards B) consists of one data card for each laminate configuration (set geometry) to be analyzed. The information needed on each card is IQ (the total number of ply sets), H (the nominal ply thickness), IP1 (the number of inner plys in each set), IPT (the number of $\pm\theta$ plys in each ply set), IP2 (the number of outer plies in each set), THETA (the ply angle associated with IPT), IPS1 (indicator for IP1), and IPS2 (an indicator for IP2). The format for phase II input is (I4, F8.0, 3I4, F12.0, 2I4). This means that the integers are IQ, IP1, IP2, IPS1, and IPS2, and the numbers requiring decimals are H and THETA. An example phase II green sheet is shown in this section following the text. Note that for the integer numbers (no decimals), the format calls for them to to be against the right side of the allowed width indicated on the sample green sheets. For integer numbers, a zero is placed by the computer wherever there is a blank on the data card.

The total number of sets indicator IQ must always be an even integer (2, 4, 6 .....) because of the assumed symmetry of the layup. The number IP1 is the total number of 0 or 90 degree plies nearest the centerline of symmetry of the laminates as shown in figure 1. It can be zero. IPT is the number of $\pm\theta$ plies in each set. For example, if IPT = 3, then there are three $\pm\theta$ pairs in each ply set (i.e., six plies). It can be zero. IP2 is like IP1 except that it is the total number of 0 or 90 degree plies in each set that are toward the outside or surface of the laminate. It also can be zero. THETA is the plus and minus angle of the middle plies. The program assumes that for an input of $+\theta$, the $+\theta$ ply starts closest to the centerline. If the opposite is desired, then the THETA value must be input as a minus THETA. IPS1 indicates the angle of the inner plies. If it is zero, then the inner plies of each set are 0 degrees, and if it is 90, then the inner plies are at 90 degrees. It cannot take any value other than 0 or 90. IPS2 is like IPS1 except that is refers to the outer plies in each set.

The input data flow chart for AC-2 is:





t^* = NOMINAL PLY THICKNESS (H)

t_s = SET THICKNESS = $t^* (_1 + 2p_\theta + _2)$

t_t = TOTAL LAMINATE THICKNESS = qt_s

q = NUMBER OF SETS (IQ)

Figure 1. AC2 Laminate Mechanical Properties Program Figure for Ply Set Orientation

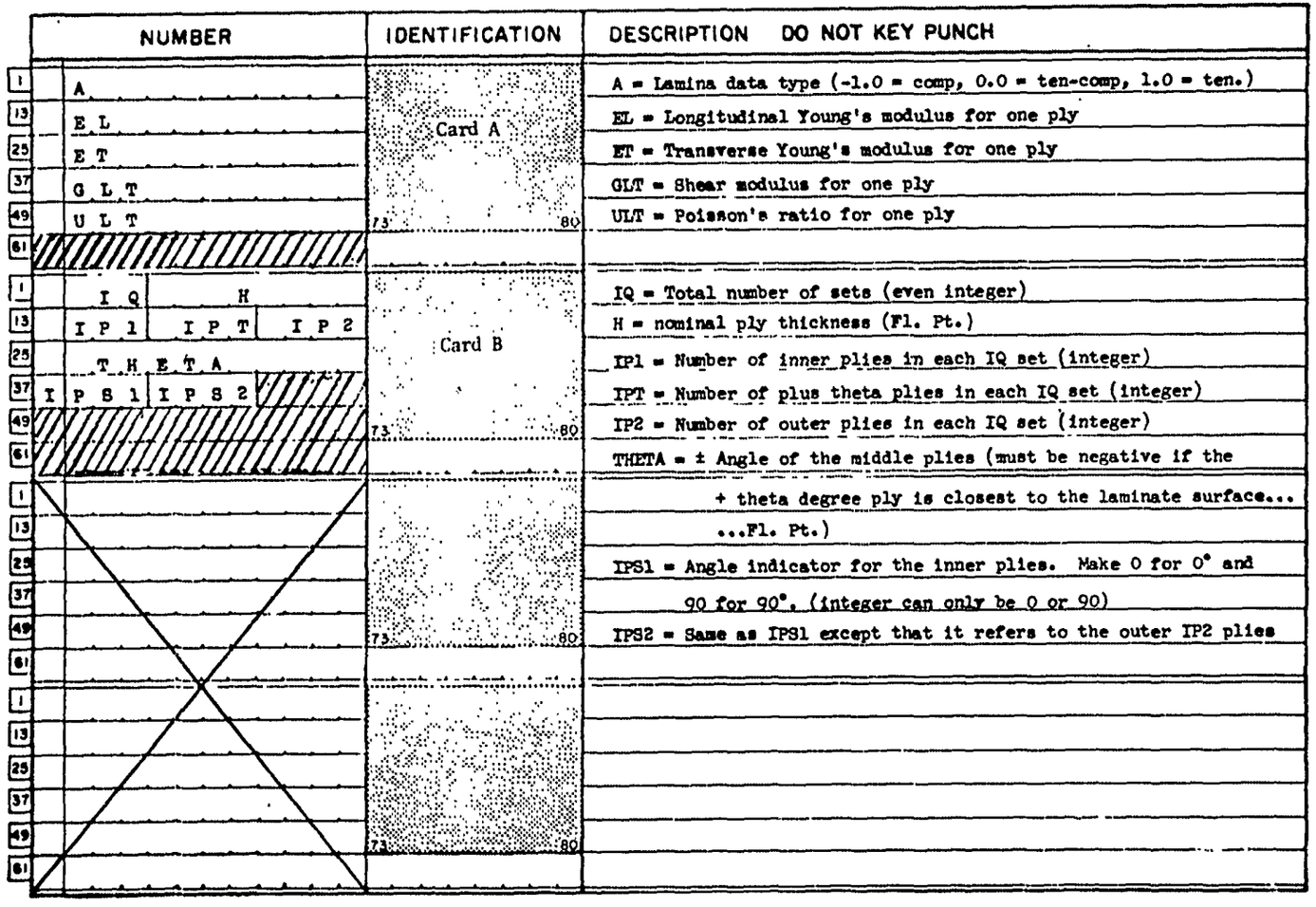
```
C
C     PROGRAM AC2 ... EXTENSIONAL AND FLEXURAL CONSTANTS FOR BALANCED,
C     SYMMETRICAL LAMINATES OF THE FORM (0/+-THETA/90)NS
C
C
C     THE INPUT DATA IS AS FOLLOWS.............
C          THE FIRST DATA CARD MUST CONTAIN THE LAMINATE ELASTIC CONSTANTS IN
C          THE FOLLOWING FORMAT...(5F12.0)
C          THE LAMINATE ELASTIC CONSTANTS ARE A,EL,ET,GLT,ULT WHERE.....
C            A IS A CONTROL CARD THAT INDICATES WHICH TYPE OF ELASTIC
C               CONSTANTS ARE TO BE USED. A=-1.0 IS FOR COMPRESSION
C               LOADINGS, A=0.0 IS FOR TENSION-COMPRESSION LOADINGS, AND
C               A=1.0 IS FOR TENSION LOADINGS.
C            EL IS THE LAMINA LONGITUDINAL YOUNGS MODULUS
C            ET IS THE LAMINA TRANSVERSE YOUNGS MODULUS
C            GLT IS THE LAMINA SHEAR MODULUS
C            ULT IS THE LAMINA POISSONS RATIO
C
C          THE SECOND AND FOLLOWING DATA CARDS MUST CONTAIN THE SET
C          DESCRIPTIONS IN THE FOLLOWING FORMAT...(I4,F8.0,3I4,F12.0,2I4)
C          THE SET DESCRIPTIONS INCLUDE Q,H,IP1,IPT,IP2,THETA,IPS1, AND IPS2
C            WHERE.....
C            IQ     IS THE TOTAL NUMBER OF SETS IN THE LAMINATE ( IT MUST ALWAYS
C               BE AN EVEN INTEGER)
C            H      IS THE NOMINAL PLY THICKNESS (FLOAT. PT.)
C            IP1    IS THE NUMBER OF INNER PLYS IN EACH IQ SET IN THE ZERO OR
C               DEGREE DIRECTION DEPENDING ON IPS1 (INTEGER)
C            IPT IS THE NUMBER OF MIDDLE PLIES IN EACH Q SET IN THE PLUS AND
C               MINUS THETA DEGREE DIRECTION (INTEGER)
C            IP2    IS THE NUMBER OF OUTER PLIES IN EACH IQ SET IN THE ZERO OR
C               DEGREE DIRECTION DEPENDING ON IPS2 (INTEGER)
C            THETA IS THE PLUS AND MINUS ANGLE OF THE P2 PLIES (FLOAT. PT.)
C            IPS1 IS AN INDICATOR TO DETERMINE IF THE ANGLES ASSOCIATED WITH
C               P1 ARE ZERO OR NINETY DEGREES. IF IPS1 IS ZERO THEN THESE ANGLES
C               ARE ZERO DEGREES. IF IPS1 IS 90 THEN THESE ANGLES ARE 90 DEGREES
C            IPS2 IS THE SAME AS IPS1 EXCEPT THAT IPS2 REFERS TO P2
C
C
C
      DIMENSION D(6),QPR(4,3),DC(3)
      READ(5, 999) A,EL,ET,GLT,ULT
  999 FORMAT(5F12.0)
 2000 READ(5,1000)  IQ,H,IP1,IPT,IP2,THETA,IPS1,IPS2
 1000 FORMAT(I4,F8.0,3I4,F12.0,2I4)
C
      IF(A)7,5,3
    3 WRITE(6,4)
    4 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TENSION LOADINGS')
      GO TO 14
    5 WRITE(6,6)
    6 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS')
      GO TO 14
    7 WRITE(6,8)
    8 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR COMPRESSION LOADINGS')
   14 WRITE(6,9) EL,ET,GLT,ULT
    9 FORMAT('0 EL='E12.5,5X'ET='E12.5,5X'GLT='E12.5,5X'MULT='F8.4///)
C
C
      Q=IQ
      P1=IP1
      PT=IPT
      P2=IP2
```

```
      RK=P1+2*PT+P2
      XN=O*RK
      R1=XN/2-RK
      R2=R1+P1
      R3=XN/2-P2
      T=XN*H
      IPT2=2*PT
C
      I=0
      PN=P1
      R=R1
  100 I=I+1
      V=O*PN/8*(2*(PN+1)*(2*PN+6*R-3*XN-2)+3*(2*R-XN-1)**2+RK*(O+2)*(3*
     1PN+6*R-3*XN+RK*(1+O))+1)
      IF(I-2) 110,120,130
  110 DC(I)=V
      PN=2*PT
      R=R2
      GO TO 100
  120 DC(I)=V
      PN=P2
      R=R3
      GO TO 100
  130 DC(I)=V
C
      C=COSD(THETA)
      S=SIND(THETA)
      C2S2=(C*S)**2
      S2=S**2
      C2=C**2
      C2TS2=C2*S2
      C4=C**4
      S4=S**4
      C4PS4=C4+S4
C
      UTL=(ET/EL)*ULT
      PSI=1-ULT*UTL
      Q11=EL/PSI
      Q12=Q11*UTL
      Q22=ET/PSI
      Q66=GLT
C
      QPR(1,2)=Q11*C4+2*(Q12+2*Q66)*C2S2+Q22*S4
      QPR(2,1)=Q12
      QPR(2,2)=(Q11+Q22-4*Q66)*C2S2+Q12*(C4+S4)
      QPR(2,3)=Q12
      QPR(3,2)=Q11*S4+2*(Q12+2*Q66)*C2S2+Q22*C4
      QPR(4,1)=Q66
      QPR(4,2)=(Q11+Q22-2*Q12-2*Q66)*C2S2+Q66*(C4+S4)
      QPR(4,3)=Q66
      IF(IPS1-90)  140,145,145
  140 QPR(1,1)=Q11
      QPR(3,1)=Q22
      GO TO 150
  145 QPR(1,1)=Q22
      QPR(3,1)=Q11
  150 IF(IPS2-90)  155,160,160
  155 QPR(1,3)=Q11
      QPR(3,3)=Q22
      GO TO 165
  160 QPR(1,3)=Q22
      QPR(3,3)=Q11
```

```
  165 CONTINUE
C
      AN=.6666666*(T/XN)**3.0
      DO 170 I=1,4
      D(I)=0.0
      DO 170 J=1,3
  170 D(I)=D(I)+QPR(I,J)*DC(J)*AN
C
      Q16T=(Q11-Q12-2*Q66)*C**3*S+(Q12-Q22+2*Q66)*C*S**3
      Q26T=(Q11-Q12-2*Q66)*C*S**3+(Q12-Q22+2*Q66)*C**3*S
C
      CON=2*Q*PT*(T/XN)**3*(-PT+XN/2-R2-RK/4*(Q+2))
      D(5)=Q16T*CON
      D(6)=Q26T*CON
C
      RATIO1=D(5)/D(1)
      RATIO2=D(5)/D(2)
      RATIO3=D(6)/D(1)
      RATIO4=D(6)/D(2)
C
      A1=P1*QPR(1,1) +2*PT*QPR(1,2) +P2*QPR(1,3)
      A12=P1*QPR(2,1) +2*PT* QPR(2,2) +P2*QPR(2,3)
      A2=P1*QPR(3,1) +2*PT*QPR(3,2) +P2*QPR(3,3)
      A6=P1*QPR(4,1) +2*PT*QPR(4,2) +QPR(4,3)*P2
C
      UYX=A12/A1
      UXY=A12/A2
      GXY=A6/RK
      EX=(A1*A2-A12**2)/A2/RK
      EY=(A1*A2-A12**2)/A1/RK
      ALF=(EX/EY)**.5
      PSII=(EX*EY)**.5/(1.0-UXY*UYX)
      B=1-UXY*UYX
      A11=EX*(1.0+EY/EX+2.0*UYX)/B
      A12=GXY-EX*UYX/B
C
      WRITE(6,575)
  575 FORMAT('0...SET DESCRIPTION')
      WRITE(6,576)IQ,H,T
  576 FORMAT('0   THERE ARE'I4  ' SETS OF PLIES WITH A NOMINAL PLY THICK
     1NESS OF'F7.4' INCHES.'/'     THE TOTAL LAMINATE THICKNESS IS'F10.4'
     2 INCHES.'/)
C
      IF(IPS1-90)  577,579,579
C
  577 WRITE(6,578)IP1
  578 FORMAT('    NUMBER OF INNER PLIES AT  0 DEGREES ='I3)
C
      GO TO 585
  579 WRITE(6,580)  IP1
  580 FORMAT('    NUMBER OF INNER PLIES AT 90 DEGREES ='I3)
C
  585 WRITE(6,590) THETA,IPT2
  590 FORMAT('    NUMBER OF MIDDLE PLIES AT (+ AND -)'F7.2' DEGREES ='I3)
C
      IF(IPS2-90)  600,620,620
C
  600 WRITE(6,610)IP2
  610 FORMAT('    NUMBER OF OUTER PLIES AT  0 DEGREES ='I3//)
      GO TO 635
C
  620 WRITE(6,630)IP2
```

```
  630 FORMAT('   NUMBER OF OUTER PLIES AT 90 DEGREES ='I3//)
C
  635 WRITE(6,640)
  640 FORMAT('0...LAMINATE FLEXURAL CONSTANTS'/)
C
      WRITE(6,650) D(1),D(2),D(3),A1,A12,D(4),D(5),D(6),A2,A6
  650 FORMAT('D11 ='E12.5,3X'D12 ='E12.5,3X'D22 ='E12.5,7X'A11 ='E12.5,
     13X'A12 ='E12.5/'D66 ='E12.5,3X'D16 ='E12.5,3X'D26 ='E12.5,7X'A22 =
     2'E12.5,3X'A66 ='E12.5//)
C
      WRITE(6,660)
  660 FORMAT('0...RATIOS BETWEEN FLEXURAL CONSTANTS'/)
C
      WRITE(6,670) RATIO1,RATIO2,RATIO3,RATIO4
  670 FORMAT('D16/D11 ='E12.5,5X'D16/D12 ='E12.5,5X'D26/D11 ='E12.5,5X'D
     126/D12 ='E12.5//)
C
      WRITE(6,680)
  680 FORMAT('0...LAMINATE EXTENSIONAL CONSTANTS')
C
      WRITE(6,13) EX,EY,GXY,UXY,UYX,ALF,PSII,AI1,AI2
   13 FORMAT('0'4X'EX='E12.5,5X'EY='E12.5,5X'GXY='E12.5,5X'MUXY='F7.4,5X
     1'MUYX='F7.4/' '1X'ALPHA=SQRT(EX/EY)='E12.5,13X'PSI=SQRT(EX*EY)/(1-
     2MUXY*MUYX)='E12.5/' '3X'AI1=EX*(1+EY/EX*2*MUYX)='E12.5,5X'AI2=GXY-
     3EX*MUYX/(1-MUXY*MUYX)='E12.5//)
C
      BC=T**3/12/8
      D11A=BC*EX
      D12A=UYX*EX*BC
      D22A=BC*EY

      D66A=GXY*8*BC
C
      WRITE(6,700)
  700 FORMAT('0...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL
     1CONSTANTS'/)
C
      WRITE(6,710)  D11A,D12A,D22A,D66A
  710 FORMAT('D11E ='E12.5,5X'D12E ='E12.5,5X'D22E ='E12.5,5X'D66E ='E12
     1.5)
C
      GO TO 2000
      END
```


NUMBER
IDENTIFICATION
DESCRIPTION DO NOT KEY PUNCH
A
E L
E T
G L T
U L T
Card A
73
80
A = Lamina data type (-1.0 = comp, 0.0 = ten-comp, 1.0 = ten.)
EL = Longitudinal Young's modulus for one ply
ET = Transverse Young's modulus for one ply
GLT = Shear modulus for one ply
ULT = Poisson's ratio for one ply
I Q
H
I P 1
I P T
I P 2
T H E T A
I P S 1
I P S 2
Card B
73
80
IQ = Total number of sets (even integer)
H = nominal ply thickness (Fl. Pt.)
IP1 = Number of inner plies in each IQ set (integer)
IPT = Number of plus theta plies in each IQ set (integer)
IP2 = Number of outer plies in each IQ set (integer)
THETA = ± Angle of the middle plies (must be negative if the
+ theta degree ply is closest to the laminate surface...
...Fl. Pt.)
IPS1 = Angle indicator for the inner plies. Make 0 for 0° and
90 for 90°. (integer can only be 0 or 90)
IPS2 = Same as IPS1 except that it refers to the outer IP2 plies
73
80
73
80

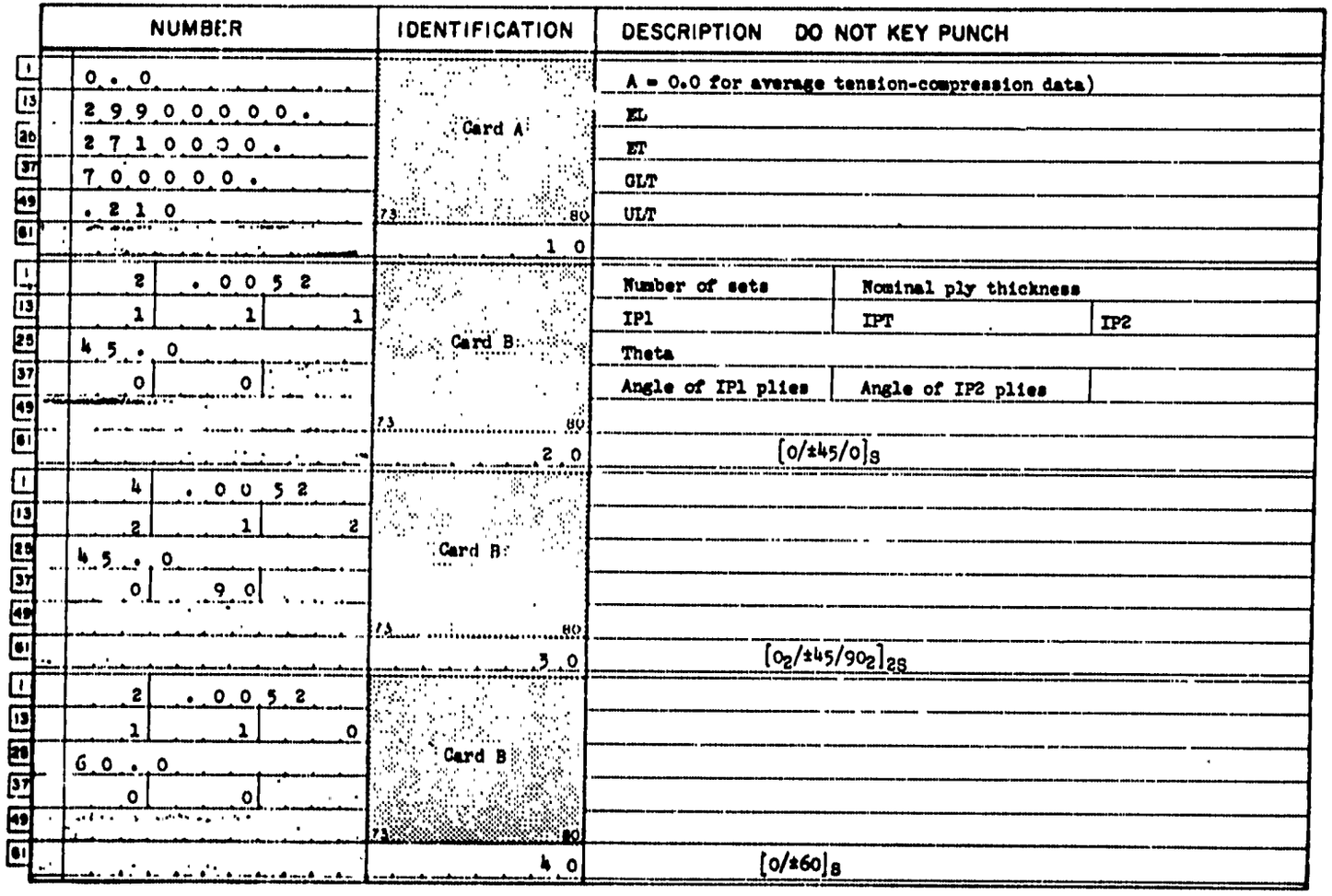
EXAMPLE PROBLEM

AC-2 LAMINATE FLEXURAL AND EXTENSIONAL PROPERTIES

Consider the three laminates: $[0/\pm45/0]_S$, $[0_2/\pm45/90_2]_S$, and $[0/\pm60]_S$. AC-2 is used to find the elastic and flexural constants from the single-ply properties which, for room temperature, are:

$$E_L = 29.9 \times 10^6 \text{ psi}$$
$$E_T = 2.71 \times 10^6 \text{ psi}$$
$$G_{LT} = 0.70 \times 10^6 \text{ psi}$$
$$\nu_{LT} = 0.210$$

The data sheets for these three laminates, as well as the output printouts, are shown at the end of this section.



NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
0.0	Card A	A = 0.0 for average tension-compression data)
29900000.		EL
2710000.		ET
700000.		GLT
.210		ULT
	10	
2 | .0052	Card B	Number of sets | Nominal ply thickness
1 | 1 | 1		IP1 | IPT | IP2
45.0		Theta
0 | 0		Angle of IP1 plies | Angle of IP2 plies
	20	$[0/\pm45/0]_S$
4 | .00522	Card B	
2 | 1 | 2		
45.0		
0 | 90		
	30	$[0_2/\pm45/90_2]_{2S}$
2 | .0052	Card B	
1 | 1 | 0		
60.0		
0 | 0		
	40	$[0/\pm60]_S$

```
...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

 EL= 0.29900E 08      ET= 0.27100E 07      GLT= 0.70000E 06      MULT=  0.2100


...SET DESCRIPTION

   THERE ARE   2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
   THE TOTAL LAMINATE THICKNESS IS    0.0416 INCHES.

  NUMBER OF INNER PLIES AT  0 DEGREES =  1
  NUMBER OF MIDDLE PLIES AT (+ AND -)  45.00 DEGREES =  2
  NUMBER OF OUTER PLIES AT  0 DEGREES =  1


...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12928E 03   D12 = 0.20975E 02   D22 = 0.32043E 02       A11 = 0.78382E 08   A12 = 0.16685E 08
D66 = 0.21746E 02   D16 =-0.76769E 01   D26 =-0.76769E 01       A22 = 0.23784E 08   A66 = 0.17199E 08


...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.59380E-01      D16/D12 =-0.36601E 00      D26/D11 =-0.59380E-01      D26/D12 =-0.36601E 00


...LAMINATE EXTENSIONAL CONSTANTS

    EX= 0.16669E 08      EY= 0.50580E 07      GXY= 0.42998E 07      MUXY= 0.7015      MUYX= 0.2129
 ALPHA=SQRT(EX/EY)= 0.18154E 01                PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.10794E 08
   A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08        A12=GXY-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06


...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.11756E 03      D12E = 0.25024E 02      D22E = 0.35671E 02      D66E = 0.25795E 02
```

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 FT= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 4 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.1248 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 2
NUMBER OF MIDDLE PLIES AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 90 DEGREES = 2

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.15476E 04 D12 = 0.45968E 03 D22 = 0.30215E 04 A11 = 0.83824E 08 A12 = 0.17827E 08
D66 = 0.48052E 03 D16 =-0.46062E 02 D26 =-0.46062E 02 A22 = 0.83824E 08 A66 = 0.18599E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.29764E-01 D16/D12 =-0.10020E 00 D26/D11 =-0.29764E-01 D26/D12 =-0.10020E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.13339E 08 EY= 0.13339E 08 GXY= 0.30998E 07 MUXY= 0.2127 MUYX= 0.2127
ALPHA=SQRT(EX/EY)= 0.10000E 01 PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.13971E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXY-FX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.22630E 04 D12E = 0.48128E 03 D22E = 0.22630E 04 D66E = 0.50211E 03

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0312 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 60.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 0

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12919E 02 D12 = 0.14606E 02 D22 = 0.43626E 02 A11 = 0.38312E 08 A12 = 0.12513E 08
D66 = 0.14932E 02 D16 =-0.31417E 01 D26 =-0.10155E 02 A22 = 0.38312E 08 A66 = 0.12899E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.24319E 00 D16/D12 =-0.21509E 00 D26/D11 =-0.78608E 00 D26/D12 =-0.69527E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.11408E 08 EY= 0.11408E 08 GXY= 0.42998E 07 MUXY= 0.3266 MUYX= 0.3266
ALPHA=SQRT(EX/EY)= 0.10000E 01 PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.12771E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXY-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.32322E 02 D12E = 0.10557E 02 D22E = 0.32322E 02 D66E = 0.10882E 02

COMPUTER PROGRAM AC-5

H/C SANDWICH PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES COMPUTER PROGRAM DECK NO. AC-5

ABSTRACT

Fortran program deck No. AC-5 is a program to calculate the general stability of a honeycomb (H/C) panel with orthotropic core and face sheets under biaxial loading. The loading ratio ϕ ($\phi = N_y/N_x$) can be zero, plus, or minus. Thus, AC-5 will handle panels under uniaxial compression, biaxial tension-compression, and biaxial compression-compression. A listing of AC-5 is shown following the text in this section.

PROGRAM INPUT DATA

The input data are shown on the sample data sheets in this section. The data, except for the title card, are read in with a 6F12.0 format per card. The definitions of the input data areas follow:

Card A:

TIT = Any desired one-card title in columns 1-72. If no title is desired, use a blank card.

Card B:

TNAØB = Total number of (a/b)'s to be considered; maximum of 20.

ALF = Alpha = $\sqrt{E_x/E_y}$ (from program AC-5).

TNN = Total number of half-waves in the y-direction; no limit.

TNM = Total number of half-waves in the x-direction; no limit.

TNPHI = Total number of load ratios ϕ; maximum of 20 (from program AC-5).

TNV = Total number of V's; maximum of 20.

Card C:

AI1 = Invariant 1 = $E_x\left(1 + E_y/E_x + 2\ \nu_{yx}\right)\Big/\left(1 - \nu_{xy}\nu_{yx}\right)$; can be found from program AC-5.

AI2 = Invariant 2 = $G_{xy} - E_x\ \nu_{yx}\Big/\left(1 - \nu_{xy}\nu_{yx}\right)$; can be found from program AC-5.

R = Orthotropic core shear modulus ratio; G'_{cx}/G'_{cy}.

TNPSI = Total number of ψ; maximum of 20.

Card(s) D:

AØB(I) = Panel aspect ratios $(a/b)_i$. There are six of these to a card up to a maximum of 20 (i.e., four cards).

Card(s) E:

PHI(I) = Load ratios, $\phi = N_y/N_x$. There are six of these to a card up to a maximum of 20. These cards are to be left out if TNPHI = 0.0.

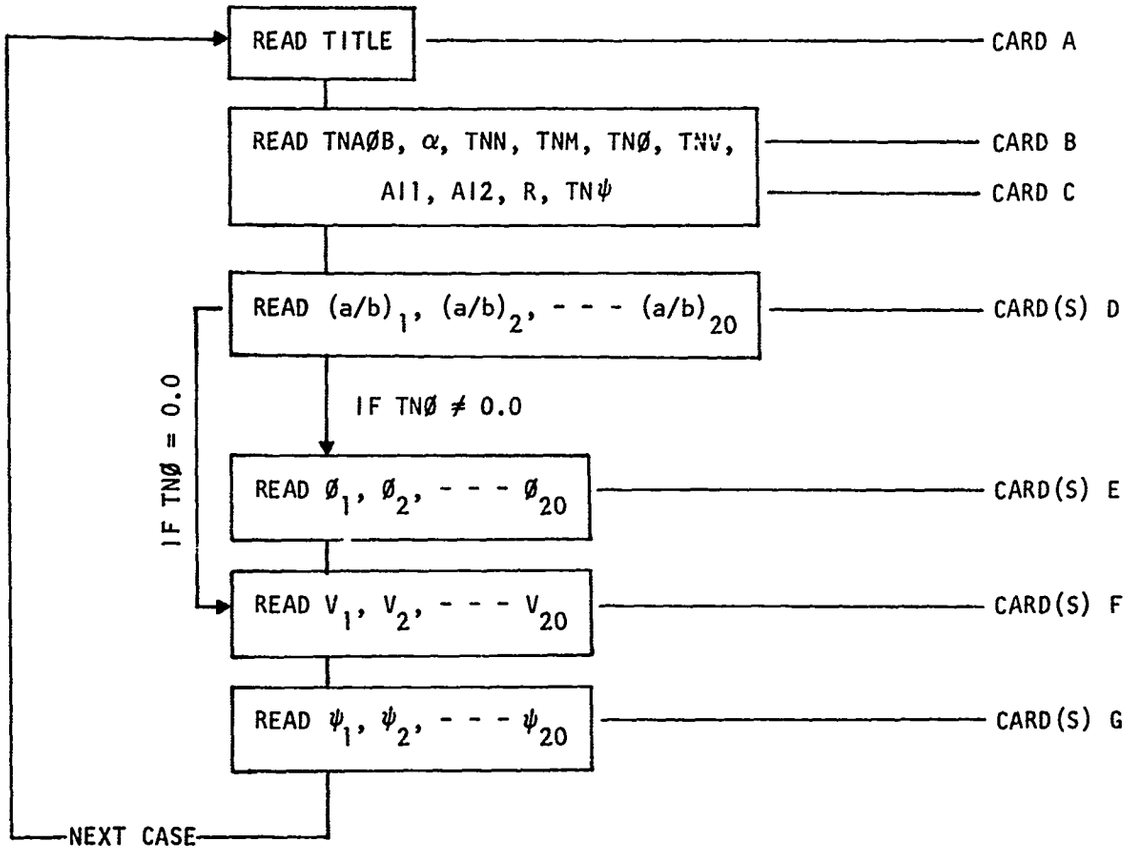
Card(s) F:

V(I) = Core shear parameters, where $V = \pi^2 \psi t_f \left(c + t_f\right)/\left(2G'_{cx} b^2\right)$. There are six of these to a card up to a maximum of 20.

Card(s) G:

PSI(I) = Modulus ratios found from AC-2 where $\psi = \sqrt{E_x E_y}/\left(1 - \nu_{xy}\nu_{yx}\right)$. There are six of these to a card up to a maximum of 20.

An input data flow chart is included. Note that additional cases are simply stacked one behind the other.



OUTPUT

The output, as shown in this section, gives the title, the core shear ratio R, gamma, beta, alpha, psi, and V. It gives k_x and k_y for uniaxial compression in the x and y directions, respectively, for zero phi (phi = N_y/N_x) and any other values of phi as given in the input data.

```
C
C     PROGRAM AC5 ... H/C PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING
C
C     THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C          AI1 = INVARIANT 1 = EX/(1-UMXY*UMYX)*(1+EY/EX+2*UMYX)
C          AI2 = INVARIANT 2 = GXY - UMYX*EX/(1-UMXY*UMYX)
C          ALF = SQRT(EX/EY)
C          BETA = B = ALF*UMYX + 2*GAMMA
C          D11 = EX*TF*C**2 /(2.*(1-UMXY*UMYX))
C          D22 = D11*EY/EX
C          D66 = TF*C**2*GXY/2
C          DOX = C*GCX
C          DOY = C*GCY
C          GAMMA = G = GXY*(1-UMXY*UMYX)/SQRT(EX*EY)
C          IA = TOTAL NUMBER OF A/B
C          IC = TOTAL NUMBER OF HALF WAVELENGTHS IN Y-DIRECTION
C          ID = TOTAL NUMBER OF HALF WAVELENGTHS IN X-DIRECTION
C          IE = TOTAL NUMBER OF PHI, WHERE PHI = NY/NX
C          IF = TOTAL NUMBER OF V WHERE V = PI**2*SQRT(D11*D22)/(B**2*DOX)
C          PSI = SQRT(EX*EY)/(1-UMXY*UMYX)
C          R = DOX/DOY
C
      DIMENSION AOB(20), PHI(20), V(20), AK(20), APSI(20), TIT(18),
     1AAK(20),AAKAY(20), AKAY(20)
    1 READ(5,100) (TIT(I),I=1,18)
  100 FORMAT(18A4)
      READ(5,110) TNAOB, ALF, TNN, TNM, TNPHI, TNV, AI1, AI2, R, TNPSI
  110 FORMAT(6F12.0)
      A=ALF
      IA=TNAOB
      IC=TNN
      ID=TNM
      READ(5,110) (AOB(I),I=1,IA)
      IE=TNPHI
      IF(IE) 108,108,105
  105 READ(5,110) (PHI(I),I=1,IE)
  108 IP=TNV
      READ(5,110) (V(I),I=1,IP)
      L2=TNPSI
      READ(5,110) (APSI(I),I=1,L2)
      DO 300 L3=1,L2
      PSI=APSI(L3)
      GAMMA= AI1/(2.*PSI) + AI2/PSI -.5*A  -.5/A
      BETA= 1.5*AI1/PSI + 2.*AI2/PSI -1.5*A -1.5/A
      B=BETA
      G=GAMMA
      WRITE(6,120)
  120 FORMAT('1    BUCKLING COEFFICIENTS FOR  SIMPLY SUPPORTED ORTHOTROP
     1IC SANDWICH PLATES -  FLAT')
      WRITE(6,100) (TIT(I),I=1,18)
      WRITE(6,130) R, G, B, A, PSI
  130 FORMAT('0'5X'G''CX/G''CY='F5.2,5X'GAMMA='F8.4,5X'BETA='F8.3,5X'ALP
     1HA='F6.3,5X'PSI='F11.0)
      DO 300 I=1,IP
      WRITE(6,135) V(I)
  135 FORMAT(14HO DATA FOR  V=F8.4)
      IF(IE) 138,140,138
  138 WRITE(6,139)  (PHI(LL),LL=1,IE)
  139 FORMAT(' '26X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'
     1PHI='F5.1)
  140 CONTINUE
```

```
      WRITE(6,142)
  142 FORMAT('0'2X'A/B'5X'KX'6X'KY'5X'KX(1)/KY(1)'4X'KX(2)/KY(2)'4X'KX(3
     1)/KY(3)'4X'KX(4)/KY(4)'4X'KX(5)/KY(5)')
      DO 300 J=1,IA
      AB1=AOB(J)
      AB2=AB1**2
      AB3=AB1**3
      AB4=AB1**4
      AAKX= 1.0E+16
      AAKY=AAKX
      IF(IE) 145,150,145
  145 DO 148 JJ=1,IF
      AAKAY(JJ)=AAKX
  148 AAK(JJ)=AAKX
  150 CONTINUE
      DO 250 M=1,ID
      OM=M
      OM2=OM**2
      OM3=OM**3
      OM4=OM2**2
      DO 250 N=1,IC
      ON=N
      ON2=ON**2
      ON3=ON*ON2
      ON4=ON2**2
      A11= (A*OM3/AB3 +OM*ON2*B/AB1)*(-1.)
      A12= 1. + OM2*A*V(I)/AB2 +ON2*G*V(I)
      A13=(-1.)*(OM*ON*V(I)*R*(B-G)/AB1)
      A21=(ON3/A +ON*OM**2*B/AB2)
      A22= A13/R
      A23= 1.+R    *(ON2/A +G*OM2/AB2)* V(I)
      A31=(-1.)*(A*OM4/AB4 +2.*B*OM2*ON2/AB2 + ON4/A)
      A32= -OM /AB1* (OM2*A/AB2 +ON2*B)*V(I)
      A33= ON*R*V(I)*(ON2/A +* OM2*B/AB2)
      AP31=(A33*(A11*A22-A12*A21) +A32*(A13*A21-A11*A23))/ (A12*A23 -A22
     1*A13)
      AP= AP31-A31
      AKX=AP/(OM2/AB2)
      AKY=AP/ON2
      IF(AKX-AAKX) 160,170,170
  160 AAKX=AKX
  170 CONTINUE
      IF(AKY-AAKY) 175,180,180
  175 AAKY=AKY
  180 CONTINUE
      IF(IF) 185,280,185
  185 DO 195 L=1,IF
      AK(L)=AP/( OM2/AB2 +ON2*PHI(L))
      IF(AK(L)-AAK(L)) 190,195,195
  190 AAK(L)=AK(L)
  195 CONTINUE
      DO 198 L=1,IF
      AKAY(L)=AK(L)*PHI(L)
      IF(AKAY(L)-AAKAY(L)) 197,198,198
  197 AAKAY(L)=AKAY(L)
  198 CONTINUE
  250 CONTINUE
  280 WRITE(6,290) AOB(J), AAKX, AAKY, (AAK(L),AAKAY(L),L=1,IF)
  290 FORMAT(' 'F6.2,2F8.2, 5(F8.2'/'F6.2 ))
  300 CONTINUE
      GO TO 1
      END
```

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T	CARD A	TIT = Any desired one-card title in columns 1-72
13			
25			
37			
49		73 80	
61			
1	T N A O B	CARD B	TNAOB = Total number of (a/b)'s to be considered
13	A L F		ALF = $\sqrt{E_x/E_y}$
25	T N N		TNN = Total number of n
37	T N M		TNM = Total number of m
49	T N P H I	73 80	TNPHI = Total number of ϕ
61	T N V		TNV = Total number of V
1	A I 1	CARD C	AI1 = Invariant 1 = $E_x\ (1 + E_y/E_x + 2\nu_{yx})/(1 - \nu_{xy}\ \nu_{yx})$
13	A I 2		AI2 = Invariant 2 = $G_{xy} - E_x\ \nu_{yx}/(1 - \nu_{xy}\ \nu_{yx})$
25	R		R = $D_{qx}/D_{qy} = G'_{cx}/G'_{cy}$
37	T N P S I		TNPSI = Total number of ψ
49		73 80	
61			
1	A O B (1)	CARD(S) D	AOB(1) = First panel aspect ratio $(a/b)_1$
13	A O B (2)		AOB(2) = Second panel aspect ratio $(a/b)_2$
25	•		•
37	•		•
49	•	73 80	•
61	•		• (Maximum of 20 (a/b)'s)

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	P H I (1)	CARD(S) E	PHI(1) = ϕ_1 $\phi = N_y/N_x$
13	P H I (2)		PHI(2) = ϕ_2
25	•		•
37	•		•
49	•	73 80	•
61	•		• (Maximum of 20 ϕ)
1	V (1)	CARD(S) F	V(1) = V_1 $V = \pi^2 \psi t_f (c+t_f)/(2G'_{cx} b^2)$
13	V (2)		V(2) = V_2
25	•		•
37	•		•
49	•	73 80	•
61	•		• (Maximum of 20 V's)
1	P S I (1)	CARD(S) G	PSI(1) = ψ_1 $\psi = \sqrt{E_x E_y}/(1 - \nu_{xy}\nu_{yx})$
13	P S I (2)		PSI(2) = ψ_2
25	•		•
37	•		•
49	•	73 80	•
61	•		• (Maximum of 20 ψ's)
1			
13			
25			
37			
49		73 80	
61			

EXAMPLE PROBLEM

AC-5 H/C PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING

The three laminates to be used in the example are $[0/\pm45/0]_S$, $\left|0_2/\pm45/90_2\right|_{2S}$, and $[0/\pm60]_S$ at room temperature. The values for α, AI1, AI2, and ψ, E_x, E_y, G_{xy}, ν_{xy}, and ν_{yx} can be found from program AC-5.

The geometry of the panel is given as

$c = 0.5$ inch, $\quad R = D_{qx}/D_{qy} = 2.0$

$a/b = 1, 2, 3$ $\quad \phi = 0, .1, .4, 1.0, 2.0, 10.0$ $\quad n, m = 1, 2, ---6$

$b = 15$

The material properties of the core are such that

$G'_{cx} = 126{,}000$ psi (8.1 lb/ft^3 core)

Now, for $[0/\pm45/0]_S$ laminate, $\quad t_f = 8 \times .0052 = 0.0416$ inch

$\alpha = 1.815$, $\quad AI1 = 33.884 \times 10^6$, $\quad AI2 = 0.12862 \times 10^6$, $\quad \psi = 10.794 \times 10^6$

$$V = \pi^2 \psi t_f \left(c + t_f\right)/2G'_{cx} b^2 = \frac{\left(9.8696\right)\left(10.794 \times 10^6\right)\left(0.0416\right)\left(0.500 + 0.0416\right)}{2\left(0.126 \times 10^{-6}\right)\left(15\right)^2}$$

$$= 0.04233$$

For $\left|0_2/\pm45/90_2\right|_{2S}$ laminate, $\quad t_f = 24 \times 0.0052 \times 0.1248$ inch

$\alpha = 1.00$, $\quad AI1 = 33.884 \times 10^6$, $\quad AI2 = 0.12862 \times 10^6$, $\quad \psi = 13.971 \times 10^6$

$$V = \frac{(9.8696)(13.971)(0.1248)(0.6248)}{2(0.126)(15)^2} = 0.18963$$

For $[0/\pm60]_S$ laminate, $\quad t_f = 6 \times 0.0052 = 0.0312$ inch

$\alpha = 1.00$, $\quad AI1 = 33.884 \times 10^6$, $\quad AI2 = 0.12862 \times 10^6$, $\quad \psi = 12.771 \times 10^6$

$$V = \frac{(9.8696)(12.771)(0.0312)(0.5312)}{2(0.126)(15)^2} = 0.03684$$

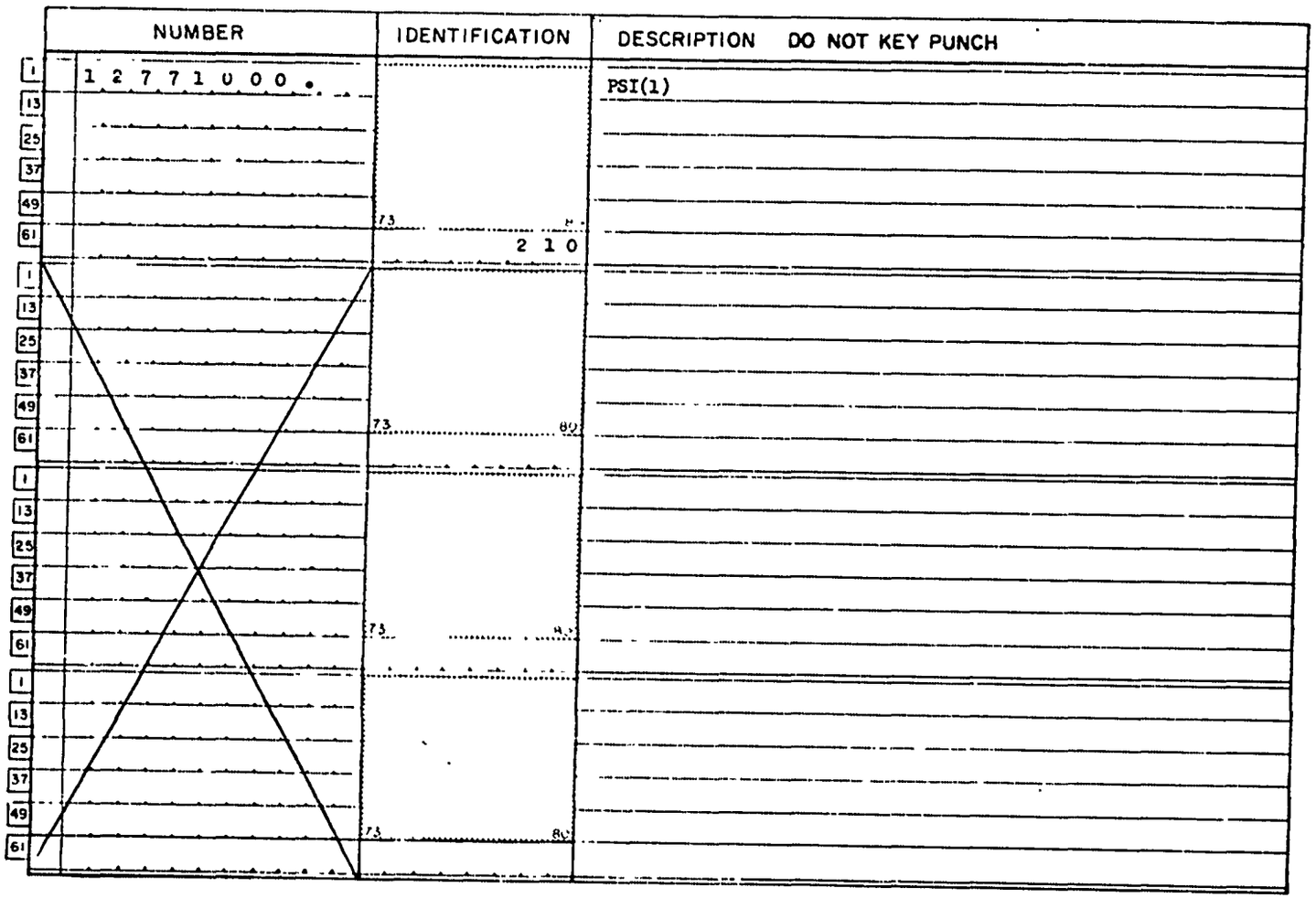
	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0/+-45/0)S		TIT [0/±45/0]$_S$ at Room Temperature
13	LAMINATE AT		
25	ROOM TEMPERA		
37	TURE		
49			
61		10	
1	3.0		TNAOB
13	1.815		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	20	TNV
1	33884000.		AI1
13	128620.		AI2
25	2.0		R
37	1.0		TNPSI
49		73 80	
61		30	
1	1.0		(a/b)$_1$
13	2.0		(a/b)$_2$
25	3.0		(a/b)$_3$
37			
49		73 80	
61		40	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.1		PHI(1)
13	0.5		PHI(2)
25	1.0		PHI(3)
37	2.0		PHI(4)
49	10.0	73 80	PHI(5)
61		50	
1	0.04233		V(1)
13			
25			
37			
49		73 80	
61		60	
1	10794000.		PSI(1)
13			
25			
37			
49		73 80	
61		70	
1	(0/0/+-45/90		TIT $[0_2/\pm45/90_2]_{2S}$ at Room Temperature
13	/90)2S LAMIN		
25	ATE AT ROOM		
37	TEMPERATURE		
49		73 80	
61		80	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3.0		TNAOB
13	1.00		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	90	TNV
1	338840000.		AI1
13	128620.		AI2
25	2.0		R
37	1.0		TNPSI
49		73 80	
61		100	
1	1.0		AOB(1)
13	2.0		AOB(2)
25	3.0		AOB(3)
37			
49		73 80	
61		110	
1	0.1		PHI(1)
13	0.5		PHI(2)
25	1.0		PHI(3)
37	5.0		PHI(4)
49	10.0	73 80	PHI(5)
61		120	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.18963		V(1)
13			
25			
37			
49		73 80	
61		130	
1	139710000.		PSI(1)
13			
25			
37			
49		73 80	
61		140	
1	(0/+-60)SLA		TIT [0/±60]$_S$ at Room Temperature
13	MINATE AT RO		
25	OM TEMPERATU		
37	RE		
49		73 80	
61		150	
1	3.0		TNAOB
13	1.0		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	160	TNV

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 3 8 8 4 0 0 0.		AI1
13	1 2 8 6 2 0.		AI2
25	2.0		R
37	1.0		TNPSI
49		73 80	
61		1 7 0	
1	1.0		AOB(1)
13	2.0		AOB(2)
25	3.0		AOB(3)
37			
49		73 80	
61		1 8 0	
1	0.1		PHI(1)
13	0.5		PHI(2)
25	1.0		PHI(3)
37	5.0		PHI(4)
49	1 0.0	73 80	PHI(5)
61		1 9 0	
1	0.03684		V(1)
13			
25			
37			
49		73 80	
61		2 0 0	


NUMBER
IDENTIFICATION
DESCRIPTION DO NOT KEY PUNCH
1 2 7 7 1 0 0 0 .
PSI(1)
73
2 1 0
80

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT
(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

DATA FOR V= 0.0423

			PHI= 0.1	PHI= 0.5	PHI= 1.0	PHI= 2.0	PHI= 10.0
A/B	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	4.17	3.91	3.79/ 0.38	2.78/ 1.39	2.09/ 2.09	1.39/ 2.78	0.38/ 3.79
2.00	4.17	1.17	3.35/ 0.34	1.56/ 0.78	0.94/ 0.94	0.52/ 1.04	0.11/ 1.14
3.00	4.06	0.79	3.31/ 0.33	1.29/ 0.65	0.71/ 0.71	0.37/ 0.75	0.08/ 0.78

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT
(0/0/+-45/90/90)2S LAMINATE AT ROOM TEMPERATURE

G'CX/G'CY= 2.00 GAMMA= 0.2219 BETA= 0.656 ALPHA= 1.000 PSI= 13971000.

DATA FOR V= 0.1896

			PHI= 0.1	PHI= 0.5	PHI= 1.0	PHI= 2.0	PHI= 10.0
A/B	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	2.27	2.19	2.06/ 0.21	1.51/ 0.76	1.13/ 1.13	0.76/ 1.51	0.21/ 2.06
2.00	2.27	1.00	2.06/ 0.21	1.33/ 0.67	0.80/ 0.80	0.44/ 0.89	0.10/ 0.97
3.00	2.27	0.84	2.06/ 0.21	1.44/ 0.67	0.75/ 0.75	0.40/ 0.79	0.08/ 0.83

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT
(0/+-60)S LAMINATE AT ROOM TEMPERATURE

G'CX/G'CY= 2.00 GAMMA= 0.4367 BETA= 1.000 ALPHA= 1.000 PSI= 12771000.

DATA FOR V= 0.0368

			PHI= 0.1	PHI= 0.5	PHI= 1.0	PHI= 2.0	PHI= 10.0
A/B	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	3.60	3.60	3.28/ 0.33	2.40/ 1.20	1.80/ 1.80	1.20/ 2.40	0.33/ 3.28
2.00	3.60	1.44	3.28/ 0.33	1.92/ 0.96	1.15/ 1.15	0.64/ 1.28	0.14/ 1.41
3.00	3.60	1.15	3.28/ 0.33	1.87/ 0.94	1.03/ 1.03	0.54/ 1.09	0.11/ 1.13

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COMPUTER PROGRAM AC-7

ANALYSIS OF SIMPLY SUPPORTED ORTHOTROPIC SANDWICH FLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-7

ABSTRACT

This program calculates the deflection, moments, and shears in a flat orthotropic honeycomb (H/C) panel under pressure plus in-plane loads, given material and geometry parameters. The method is a series summation, and the series terms as well as the series sum are printed out to enable the user to monitor convergence.

The in-plane loads can be compression-compression, tension-compression, or tension-tension. For the first two cases, part of the input data, the buckling coefficients, can be found from program AC-5. For the latter case, tension-tension buckling coefficients are found as noted later in this program description.

PROGRAM INPUT DATA

The program uses as input the results of programs AC-2 and AC-5, as well as other items. The input items are:

Card A:

TIT = Any desired one-card title. The title is in columns 1-72. If no title is desired, a blank card must be used.

Card B:

TNAØB = Total number of (a/b)'s to be read in (maximum of 20).

ALF = The alpha volume from program AC-2 $\left(\alpha = \sqrt{Ex/Ey}\right)$.

TNM = Total number of half-waves in the X-direction (no limit).

TNV = Total number of core shear influence parameters V to be read in.

AI1 = Invariant 1 from program AC-2. It is $E_x\left(1 + E_y/E_x + 2\nu_{yx}\right)/\left(1 - \nu_{xy}\nu_{yx}\right)$.

AI2 = Invariant 2 from program AC-2. It is $G_{xy} - E_x\nu_{yx}/\left(1 - \nu_{xy}\nu_{yx}\right)$.

Card C:

R_x = Ratio of N_x to critical N_x $\left(N_x/N_{x,\,cr}\right)$; plus for compression and minus for tension.

R_y = Ratio of N_y to critical N_y $\left(N_y/N_{y,\,cr}\right)$; plus for compression and minus for tension.

R = Core shear modulus ratio G'_{cx}/G'_{cy}.

TNPSI = Total number of ψ to be considered.

TNXØA = Total number of x/a to be considered.

TNYØB = Total number of y/b to be considered.

Card(s) D:

AØB(1) = Panel aspect ratio $(a/b)_1$.

AAKX(1) = Buckling coefficient K_x from:

1. AC-5 corresponding to given value of Φ if N_x is compression

2. $K_x = N_x\left(\frac{b}{\pi}\right)^2\Big/\sqrt{D_{11}\,D_{22}}$ if N_x is tension.

AAKY(1) = Buckling coefficient K_y from:

1. AC-5 corresponding to given value of Φ if N_y is compression.

2. $K_y = N_y\left(\frac{b}{\pi}\right)^2\Big/\sqrt{D_{11}\,D_{22}}$ if N_y is tension.

There are two sets of these parameters to a card (maximum of 20 sets).

Card(s) E:

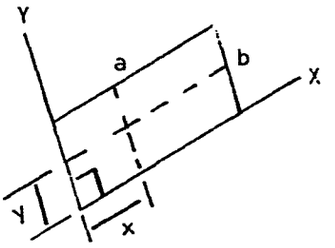
V(I) = Core shear parameters where $V = \pi^2\psi t_f\left(c + t_f\right)/\left(2G'_{cx}b^2\right)$. There are six of these to a card (maximum of 20).

Card(S) F:

APSI(I) = Modulus ratios ψ found from AC-2, where $\psi = \sqrt{E_x E_y}/\left(1 - \nu_{xy}\nu_{yx}\right)$. There are six of these to a card (maximum of 20).

Card(S) G:

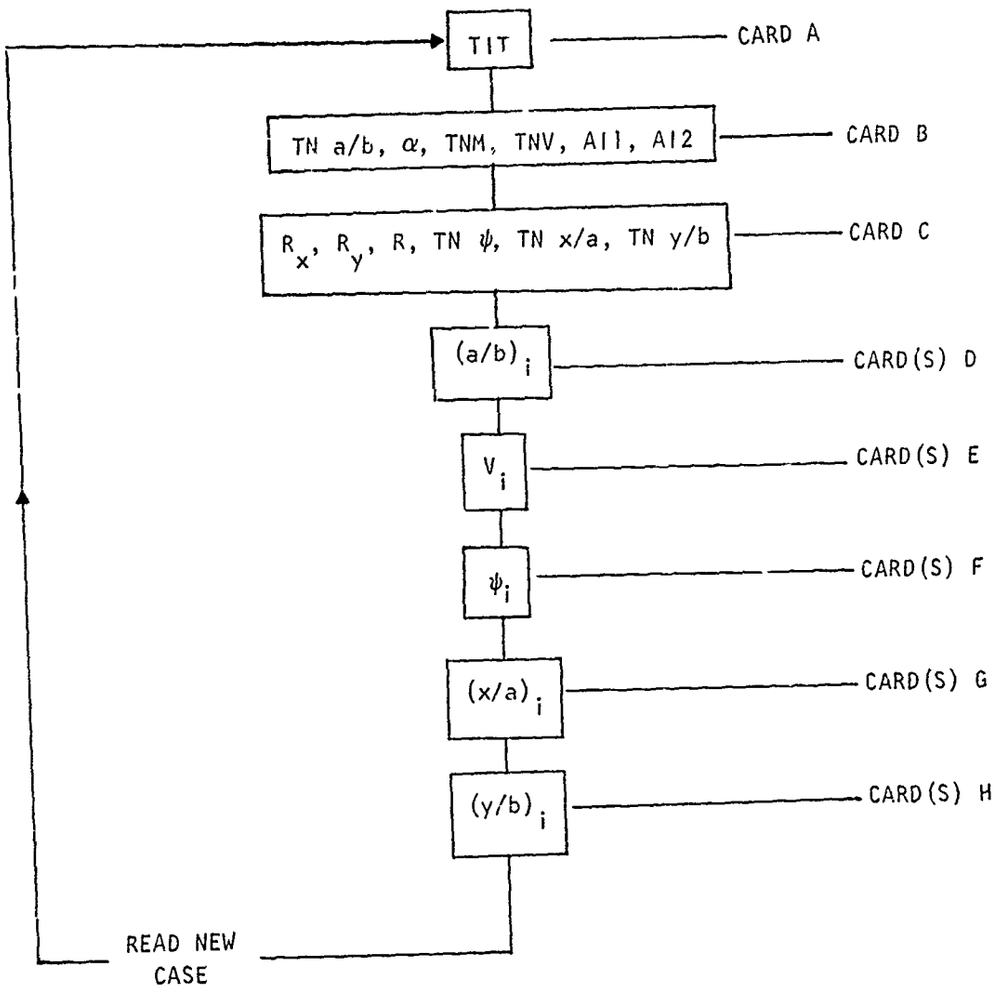
XØA(I) = Ratio of point on X-axis where results are to be given to the length of the panel, a, in the x-direction. There are six of these to a card (maximum of 20).



Card(s) H:

YØB(I) = Ratio of point on Y-axis where results are to be given to the width of the panel, b, in the y-direction. There are six of these to a card (maximum of 20).

Sample input data sheets are shown in this section following the text.



additional cases may be stacked one behind the other.

OUTPUT

The output, shown later in this section, gives the title and the parameters R, R_X, R_Y, K_X, K_Y, a/b, γ, β, α, and ψ. The series terms are printed for m = n = 1, 3, 5, - - - TNM for the values of x/a and y/b given. The last term in the set can be used to find the various quantities, where:

1. Panel deflection at x/a and y/b

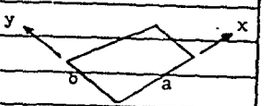
   W = (Col 1 value). $qb^4/\sqrt{D_{11} D_{22}}$

```
C
C     PROGRAM AC7 ... ANALYSIS OF SIMPLY-SUPPORTED ORTHOTROPIC SANDWICH
C     FLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS
C
C     THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.........
C          AI1 = INVARIANT 1 = EX/(1+EY/EX+2*MUYX)/(1-MUXY*MUYX)
C          AI2 = INVARIANT 2 = GXY - MUYX*EX/(1-MUXY*MUYX)
C          AKX = BUCKLING COEFFICIENT IN THE X-DIRECTION FOUND FROM PROGRAM AC5
C          AKY = BUCKLING COEFFICIENT IN THE Y-DIRECTION FOUND FROM PROGRAM AC5
C          ALPHA = ALF = SQRT(EX/EY)
C          BETA = B = ALPHA*MUYX + 2*GAMMA
C          D11 = EX*TH**3/(2*(1-MUXY*MUYX))
C          D22 = D11*(EY/EX)
C          D66 = TH**3*GXY/2
C          DOX = C*GCX
C          DOY = C*GCY
C          GAMMA = G = GXY*(1-MUXY*MUYX)/SQRT(EX*EY)
C          IA = TOTAL NUMBER OF A/B(PANEL ASPECT RATIOS)
C          II = TOTAL NUMBER OF PHI, WHERE PHI = MY/MX
C          IP = TOTAL NUMBER OF V, WHERE V = PI**2*SQRT(D11*D22)/(EX**2*DOX)
C          NI = TRUNCATION LIMIT FOR SERIES SUMMATION
C          PSI = SQRT(EX*EY)/(1-MUXY*MUYX)
C          R = DOX/DOY = GCX/GCY
C          RX = MX/MXCR
C          RY = MY/MYCR
C          XI = SQRT(D11*D22)
C
      DIMENSION AOB(20),XOA(20),YOB(20),PHI(20),V(20),QX(20),QY(20),
     1 AMX(20),AMY(20),AMXY(20),U(20),AQX(20),AQY(20),SY(20),CY(20)
     2,APSI(20),TIT(18),AAKX(20),AAKY(20)
    1 READ(5,7) (TITLE,I=1,18)
    7 FORMAT(18A4)
      READ(5,2)TNAOB,ALF,TNB,TNV,AI1,AI2,RX,RY,R,TNPSI,TNXOA,TNYOB
    2 FORMAT(6F12.0)
      A=ALF
      D1=0.0166424
      IA=TNAOB
      NI=TNB
      MA=TNXOA
      MB=TNYOB
    3 READ(5,2)  (AOB(I),AAKX(I),AAKY(I),I=1,IA)
    6 IP=TNV
      READ(5,2)(V(I),I=1,IP)
      L2=TNPSI
      READ(5,2)(APSI(I),I=1,L2)
      READ(5,2)(XOA(I),I=1,MA)
      READ(5,2)(YOB(I),I=1,MB)
      DO 100 L3=1,L2
      DO 100 J=1,IA
      AB1=AOB(J)
      AKX=AAKX(J)
      AKY=AAKY(J)
      AB2=AB1**2
      AB3=AB1**3
      AB4=AB1**4
      PSI=APSI(L3)
      GAMMA=(AI1/(2.*PSI) + AI2/PSI -.5*A  -.5/A)
      BETA=(1.5*AI1/PSI + 2.*AI2/PSI -1.5*A -1.5/A)
      B=BETA
      G=GAMMA
      WRITE(6,8)
    8 FORMAT('1')
```

```
      WRITE(6,7) (TIT(I),I=1,18)
   10 WRITE(6,11)
   11 FORMAT('0   DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SU
     1PPORTED ORTHOTROPIC  SANDWICH PLATES')
      WRITE(6,16)P,G,B,A,PSI
   16 FORMAT('0',5X,'G',7X,'GX/G',7X,'GY=',F5.2,5X,'GAMMA=',F8.4,5X,'BETA=',F8.3,5X,'ALP
     1HA=',F6.3,5X,'PSI=',F11.0)
      WRITE(6,18) AOB(J),RX,RY,AKX,AKY
   18 FORMAT('0',5X,'A/B =',F5.2,5X,'NX/NXCR =',F6.3,5X,'NY/NYCR =',F6.3,5X,'KX
     1=',F5.2,5X,'KY =',F5.2)
      DO 100 I=1,IP
      WRITE(6,27)V(I)
   27 FORMAT('0',2X,'...DATA FOR V =',F8.4)
      DO 100 J1=1,MA
      C1=3.1416*XOA(J1)
  250 WRITE(6,251)XOA(J1)
  251 FORMAT(25H     COEFFICIENTS FOR X/A=F6.3 )
  505 WRITE(6,506)
  506 FORMAT('0                    W*X1/Q*B**4       MX/Q*B**2       MY/Q*B**2     M
     1XY/Q*B**2      QX/Q*B      QY/Q*B')
      DO 199 J2=1,MB
      U(J2)=.0
      AMX(J2)=.0
      AMY(J2)=.0
      AMXY(J2)=.0
      AQX(J2)=.0
      AQY(J2)=.0
  199 CONTINUE
   30 DO 90 M=1,MI,2
      DO 90 N=1,M,2
      ON=N
      OM=M
      S1=SIN(OM*C1)
      C1=COS(OM*C1)
      DO 200 K=1,MB
      SY(K)=SIN(YOB(K)*3.1416*OM)
      CY(K)=COS(YOB(K)*3.1416*OM)
  200 CONTINUE
      OM2=OM**2
      ON2=ON**2
      OM3=OM*OM2
      OM4=OM2**2
      ON4=ON2**2
      AB1=(A*OM3/AB3+OM*ON2*B/AB1)*(-1.)
      A12=1.+OM2*A*V(I)/AK2+OM2*G*V(I)
      A13=(-1.)*(OM*ON*V(I)*R*(R-G)/AB1)
      A21=(OM3/A+OM*ON2*R/AB2)
      A22=A13/P
      A23=1.+P*(ON2/A+G*OM2/AK2)*V(I)
      A31=(-1.)*(A*OM4/AK4+2.*R*OM2*ON2/AB2+ON4/A)
      A32=-(OM2/AK2+(OM2*A/AK2+ON2*R)*V(I))
      A33=ON*P*V(I)*(OM2/A+OM2*R/AB2)
      AMN=(A12*A23-A22*A13)/(A33*(A11*A22-A12*A21)+A32*(A13*A21-A11*A23)
     1+(-A31-AKX*PX*OM2/AB2-AKY*PY*ON2)*(A12*A23-A22*A13))*Q1/(OM*ON)
      DQ=A22*A13-A12*A23
      ALF2=(A11*A22-A12*A21)/DQ
      ALF1=(A11*A23-A13*A21)/DQ
      S=V(I)
      BMX=-9.8696*AMN*((OM*S*ALF1/AB1-OM2/AB2)*A+(R-2.*G)*(R*S*
     1ON*ALF2-ON2))
      BMY=-9.8696*AMN*(1./A*(ON*ALF2*R*S-ON2)+(R-2.*G)*(ALF1*OM
```

```
     1*S/AB1-ON**2/AB2) )
      BMXY=G* 9.8696     *AMN*(ON*ALF1*S +ALF2*R*OM*S -2.*OM*ON/AB1)
      BOX=- 31.0065    *AMN*OM*ON*(1./((ON*AB1)*((B-2.*G)*(ON*ALF2*R*S
     1-ON2)+A*(ALF1*OM*S/AB1 -OM2/AB2)) +G*ON*(ALF1*S/OM +ALF2*R*S/(ON*
     2AB1) -2./AB1))
      BOY=- 31.0065    *AMN*    (G*OM*(ALF1*S/OM+ALF2*R*S/(ON*AB1)-2./AB1)
     1*(1./AB1)+(1./OM)*((ON*ALF2*R*S-ON2)/A+(B-2.*G)*(ALF1*OM*S/AB1
     2-OM2/AB2)))*OM*ON
      DO 300 J2=1,MB
      AMX(J2)=AMX(J2)+BMX*SX*SY(J2)
      AMY(J2)=AMY(J2)+BMY*SX*SY(J2)
      AMXY(J2)=AMXY(J2)+BMXY*CX*CY(J2)
      AOX(J2)=AOX(J2)+BOX*CX*SY(J2)
      AOY(J2)=AOY(J2)+BOY*SX*CY(J2)
      W(J2)=W(J2)+AMN*    SX*SY(J2)
  300 CONTINUE
      OM1=M
      IF(OM1-OM)90 ,400,90
C
C.....LOOP TO PICK UP ANY TERM
C
  400 IF(M-N)401,403,401
  401 OM=M
      ON=N
      GO TO 352
  403 CONTINUE
  500 WRITE(6,501) M,N
  501 FORMAT(4H0 M=,I2,3X,2HN=,I2)
  510 DO 520 J2=1,MB
  511 WRITE(6,512)YOB(J2),W(J2),AMX(J2),AMY(J2),AMXY(J2),AOX(J2),AOY(J2)
  512 FORMAT(5H0Y/B=F6.3,F9.6,F13.3,F14.3,F14.3,F13.3,F11.3)
  520 CONTINUE
   90 CONTINUE
  100 CONTINUE
      GO TO 1
      END
```

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T	Card A	TIT = Any desired one-card title, cols. 1-72
13			
25			
37			
49			
61		73 80	
1	T N A Ø B	Card B	TNAØB = Total number of a/b to be considered
13	A L F		ALF = Alpha = $\sqrt{E_x/E_y}$
25	T N M		TNM = Total number of M
37	T N V		TNV = Total number of V
49	A I 1		AI1 = Invariant 1 = $E_x\,(1 + E_y/E_x + 2\nu_{yx})/(1 - \nu_{xy}\nu_{yx})$
61	A I 2	73 80	AI2 = Invariant 2 = $G_{xy} - E_x\nu_{yx}/(1 - \nu_{xy}\nu_{yx})$
1	R X	Card C	RX = N_x/N_{xcr}
13	R Y		RY = N_y/N_{ycr}
25	R		R = G'_{cx}/G'_{cy}
37	T N P S I		TNPSI = Total number of ψ
49	T N X Ø A		TNXØA = Total number of x/a
61	T N X Ø B	73 80	TNYØB = Total number of y/b
1	A Ø B C (1)	Card(s) D	a/b
13	A A K X (1)		k_x (1) k_x can be found from AC5
25	A A K Y (1)		k_y k_y can be found from AC5
37	A Ø B (2)		a/b
49	A A K X (2)		k_x (2)
61	A A K Y (2)	73 80	k_y (20 maximum)



	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	V (1)		
13	V (2)		
25	V (3)	Card(s) E	$V = \pi^2 4\psi t_c \left(C + t_c\right) \left(2G_{cx}^{1} b^2\right)$
37			
49		73 8	
61			(20 Maximum)
1	A P S I (1)		
13	A P S I (2)		$\psi = \sqrt{E_x E_y} \Big/ \left(1 - \nu_{xy} \nu_{yx}\right)$
25	A P S I (3)	Card(s) F	
37			
49		73 80	
61			(20 Maximum)
1	X Ø A (1)		
13	X Ø A (2)		
25	X Ø A (3)	Card(s) G	
37			
49		73 80	
61			(20 Maximum)
1	Y Ø B (1)		
13	Y Ø B (2)		
25	Y Ø B (3)	Card(s) H	
37			
49		73	
61			(20 Maximum)

where

$$D_{11} = E_x t_f c^2 / \left| 2 \left(1 - \nu_{xy} \nu_{yx} \right) \right|$$

$$D_{22} = D_{11} E_y / E_x$$

q = pressure

2. Panel moment M_x at x/a and y/b

M_x = (Col 2 value) . q b^2

3. Panel moment M_y at x/a and y/b

M_y = (Col 3 value). q b^2

4. Panel moment M_{xy} at x/a and y/b

M_{xy} = (Col 4 value). q b^2

5. Panel shear Q_x at x/a and y/b

Q_x = (Col 5 value). q b

6. Panel shear Q_y at x/a and y/b

Q_y = (Col 6 value). q b

EXAMPLE PROBLEM

An example problem is given to illustrate the use of the program. A honeycomb panel with c = 0.5 and $[0/\pm45/0]_S$ face sheets at room temperature is loaded under pressure only and then with $N_y/N_x = 1.0$ for $N_x/N_{x,cr} = N_y/N_{y,cr} = 0.5$.

Many of the parameters on the data sheets shown are taken from the AC-5 example problem. The results are tabulated as follows:

Case	a/b	x/a	y/b	w	M_x	M_y	M_{xy}	Q_x	Q_y
Pressure only	1	0	0	0	0	0	$-0.032\ qb^2$	0	0
		0	0.5	0	0	0	0	0.5 qb	0
		0.5	0	0	0	0	0	0	0.188 qb
		0.5	0.5	$0.003835\ q\frac{b^4}{x}$	$0.069\ qb^2$	$0.028\ qb^2$	0	0	0
	2	0	0	0	0	0	$-0.056\ qb^2$	0	0
		0	0.5	0	0	0	0	0.461 qb	0
		0.5	0	0	0	0	0	0	0.394 qb
		0.5	0.5	$0.01366\ q\frac{b^4}{x}$	$0.094\ qb^2$	$0.077\ qb^2$	0	0	0
	3	0	0	0	0	0	$-0.059\ qb^2$	0	0
		0	0.5	0	0	0	0	0.459 qb	0
		0.5	0	0	0	0	0	0	0.449 qb
		0.5	0.5	$0.01976\ q\frac{b^4}{x}$	$0.094\ qb^2$	$0.104\ qb^2$	0	0	0

Case	a/b	x/a	y/b	w	M_x	M_y	M_{xy}	Q_x	Q_y
Pressure plus N_x and N_y	1	0	0	0	0	0	$-0.060\ qb^2$	0	0
		0	0.5	0	0	0	0	0.705 qb	0
		0.5	0	0	0	0	0	0	0.449 qb
		0.5	0.5	$0.007795\ q\frac{b^4}{x}$	$0.145\ qb^2$	$0.059\ qb^2$	0	0	0
	2	0	0	0	0	0	$-0.107\ qb^2$	0	0
		0	0.5	0	0	0	0	0.811 qb	0
		0.5	0	0	0	0	0	0	0.457 qb
		0.5	0.5	$0.02781\ q\frac{b^4}{x}$	$0.202\ qb^2$	$0.160\ qb^2$	0	0	0
	3	0	0	0	0	0	$-0.110\ qb^2$	0	0
		0	0.5	0	0	0	0	0.769 qb	0
		0.5	0	0	0	0	0	0	0.864 qb
		0.5	0.5	$0.04045\ q\frac{b^4}{x}$	$0.204\ qb^2$	$0.218\ qb^2$	0	0	0

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0/+-45/0).S.		TIT
13	LAMINATE AT		
25	RØØM TEMPERA		Start of First Case
37	TURE, PRESSU		
49	RE ØNLY	73 80	
61		10	
1	3.0		TNAØB
13	1.815		ALF
25	1.0 0		TNM
37	1.0		TNV
49	3388400 0	73 5	AI1
61	12862 0	20	AI2
1	0.0		RX
13	0.0		RY
25	2.0		R
37	1.0		TNPSI
49	2.0	73 90	TNXØA
61	2.0	30	TNYØB
1	1.0		AØB(1)
13	4.17		AAKX(1)
25	3.91		AAKY(1)
37	2.0		AØB(2)
49	4.17	73	AAKX(2)
6.	1.17	40	AAKY(2)

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3.0		AØB(3)
13	4.06		AAKX(3)
25	0.79		AAKY(3)
37			
49		73	80
61			50
1	0.04233		V(1)
13			
25			
37			
49		73	80
61			60
1	10794000.		APSI(1)
13			
25			
37			
49		73	80
61			70
1	0.0		XØA(1)
13	0.5		XØA(2)
25			
37			
49		73	80
61			80

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.0		YØB(1)
13	0.5		YØB(2)
25			
37			
49		73 8	
61		90	
1	(0/+-4510)S		TIT
13	LAMINATE AT		
25	RØØM TEMPERA		Start of Second Case
37	TURE, UNDER		
49	PRESSURE PLU	73 9	
61	S NX AND NY	100	
1	3.0		THAØB
13	1.815		ALF
25	10.0		TNH
37	1.0		TNV
49	33840000	73 8	AI1
61	128620.	110	AI2
1	0.5		RX
13	0.5		RY
25	2.0		P
37	1.0		TNPSI
49	2.0	73	TNXJA
61	2.0	120	TNYØB

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1.0		AØB(1)
13	2.09		AAKX(1)
25	2.09		AAKY(1)
37	2.0		AØB(2)
49	0.94	73 80	AAKX(2)
61	0.94	130	AAKY(2)
1	3.0		AØB(3)
13	0.71		AAKX(3)
25	0.71		AAKY(3)
37			
49			
61		73 80	
		140	
1	0.042 33		V(1)
13			
25			
37			
49			
61		73 80	
		150	
1	10794000.		APSI(1)
13			
25			
37			
49			
61		73 80	
		160	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.0		XØA(1)
13	0.5		XØA(2)
25			
37			
49		73 80	
61		170	
1	0.0		YØB(1)
13	0.5		YØB(2)
25			
37			
49		73 80	
61		180	
1			
13			
25			
37			
49		73 80	
61			
1			
13			
25			
37			
49		73	
61			

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 1.00 . NX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.17 KY = 3.91

...DATA FOR V = 0.0423
COEFFICIENTS FOR X/A= 0.0

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.028	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.328	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.337	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.367	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.368	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.032	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.378	0.0

COEFFICIENTS FOR X/A= 0.500

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.188
Y/B= 0.500	0.003988	0.077	0.032	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.218
Y/B= 0.500	0.003816	0.068	0.027	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.238
Y/B= 0.500	0.003839	0.070	0.028	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.244
Y/B= 0.500	0.003832	0.069	0.028	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.251
Y/B= 0.500	0.003835	0.069	0.028	-0.000	-0.000	-0.000

[0/+-45/0]S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.4985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 2.00 MX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.17 KY = 1.17

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W=XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q=B	QY/Q=B
M= 1 N= 1						
Y/B= 0.0 0.0	0.0	0.0	-0.050	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.336	0.0	
M= 3 N= 3						
Y/B= 0.0 0.0	0.0	0.0	-0.055	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.395	0.0	
M= 5 N= 5						
Y/B= 0.0 0.0	0.0	0.0	-0.056	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.436	0.0	
M= 7 N= 7						
Y/B= 0.0 0.0	0.0	0.0	-0.056	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.448	0.0	
M= 9 N= 9						
Y/B= 0.0 0.0	0.0	0.0	-0.056	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.461	0.0	

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.348
Y/B= 0.500	0.014188	0.110	0.085	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.355
Y/B= 0.500	0.013603	0.091	0.075	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.384
Y/B= 0.500	0.013671	0.095	0.077	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.385
Y/B= 0.500	0.013653	0.094	0.076	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.394
Y/B= 0.500	0.013660	0.094	0.077	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.415 PSI= 10794000.

A/B = 3.00 MX/MXCR = 0.0 NY/MYCR = 0.0 KX = 4.06 KY = 0.79

...DATA FOR V = 0.0423
COEFFICIENTS FOR X/A= 0.0

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0 0.0	0.0	0.0	-0.049	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.285	0.0	
M= 3 N= 3						
Y/B= 0.0 0.0	0.0	0.0	-0.058	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.370	0.0	
M= 5 N= 5						
Y/B= 0.0 0.0	0.0	0.0	-0.059	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.421	0.0	
M= 7 N= 7						
Y/B= 0.0 0.0	0.0	0.0	-0.059	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.447	0.0	
M= 9 N= 9						
Y/B= 0.0 0.0	0.0	0.0	-0.059	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.459	0.0	

COEFFICIENTS FOR X/A= 0.500

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.421
Y/B= 0.500	0.021051	0.116	0.117	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.399
Y/B= 0.500	0.019594	0.087	0.101	-0.000	0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.442
Y/B= 0.500	0.019796	0.096	0.105	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.436
Y/B= 0.500	0.019745	0.092	0.103	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.449
Y/B= 0.500	0.019763	0.094	0.104	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 1.00 NX/NXCR = 0.500 NY/NYCR = 0.500 KX = 2.09 KY = 2.09

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.055	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.656	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.059	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.661	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.694	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.695	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.705	0.0

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.376
Y/B= 0.500	0.007989	0.154	0.065	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.419
Y/B= 0.500	0.007774	0.143	0.058	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.443
Y/B= 0.500	0.007800	0.145	0.060	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.450
Y/B= 0.500	0.007792	0.144	0.059	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.457
Y/B= 0.500	0.007795	0.145	0.059	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 2.00 NX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.94 KY = 0.94

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0 0.0	0.0	0.0	-0.101	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.674	0.0	
M= 3 N= 3						
Y/B= 0.0 0.0	0.0	0.0	-0.107	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.742	0.0	
M= 5 N= 5						
Y/B= 0.0 0.0	0.0	0.0	-0.107	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.785	0.0	
M= 7 N= 7						
Y/B= 0.0 0.0	0.0	0.0	-0.107	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.798	0.0	
M= 9 N= 9						
Y/B= 0.0 0.0	0.0	0.0	-0.107	0.0	0.0	
Y/B= 0.500 0.0	0.0	0.0	0.000	0.811	0.0	

COEFFICIENTS FOR X/A= 0.500

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.697
Y/B= 0.500	0.028423	0.220	0.170	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.705
Y/B= 0.500	0.027754	0.198	0.159	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.737
Y/B= 0.500	0.027826	0.203	0.161	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.738
Y/B= 0.500	0.027807	0.201	0.160	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.738
Y/B= 0.500	0.027814	0.202	0.160	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 3.00 MX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.71 KY = 0.71

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.098	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.568	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.108	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.674	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.729	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.751	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.769	0.0

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.840
Y/B= 0.500	0.042011	0.231	0.233	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.840
Y/B= 0.500	0.040266	0.197	0.215	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.856
Y/B= 0.500	0.040487	0.206	0.218	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.850
Y/B= 0.500	0.040432	0.202	0.217	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.864
Y/B= 0.500	0.040451	0.204	0.218	-0.000	-0.000	-0.000

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COMPUTER PROGRAM AC-11

H/C SANDWICH PANEL STABILITY FOR IN-PLANE SHEAR LOADING

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES COMPUTER PROGRAM DECK NO. AC-11

ABSTRACT

This program calculates the general buckling load for a flat honeycomb (H/C) panel with orthotropic face sheets and core under in-plane shear. The program listing includes a modified eigenvalue solution subroutine which involves convergence of the geometric mean of succeeding eigenvalues. The face-sheet thicknesses are assumed to be equal.

PROGRAM INPUT DATA

The input data can be of two types, material property input or material parameter input. The input data format is all 6F12.0 except for the title card. Sample data sheets are included following the text in this section. The data description is as follows:

Card A:

TIT = any desired one-card title. The title is in columns 1-72. If no title is desired, a blank card must be used.

Card B:

CNTL = Input data form indicator which is 0.0 for material property input (cards C_o and D_o) and 1.0 for material parameter input (cards C_1 and D_1).

Q = Range of m and n (maximum of 13.0).

Card C_o: Used if CNTL = 0.0.

EX = Young's modulus in the x-direction.

EY = Young's modulus in the y-direction.

GXY = Shear modulus.

UMXY = Poisson's ratio ν_{xy}.

GCX = Core shear modulus in the x-direction (G'_{cx}).

GCY = Core shear modulus in the y-direction (G'_{cy}).

Card D_o: Used if CNTL = 0.0.

A = Length of panel in the x-direction, a.

B = Width of panel in the y-direction, b.

C = Core depth, c.

TF = One face-sheet thickness, t_f.

XNIT = Maximum number of iterations to be allowed (usually set at 100.0).

Card C_1: Used if CNTL = 1.0.

ALPHA = The alpha value from program AC-2 $\left(\alpha = \sqrt{E_x/E_y}\right)$.

PSI = Modulus ratio ψ found from AC-2 where $\psi = \sqrt{E_x E_y} \Big/ \left(1 - \nu_{xy}\nu_{yx}\right)$.

V = Core shear parameters where $V = \pi^2 \psi t_f (c + t_f) \Big/ \left(2G'_{cx} b^2\right)$.

R = Core shear modulus ratio = G'_{cx}/G'_{cy}.

AI1 = Invariant 1 from AC-2. $I_1 = E_x\left(1 + E_y/E_x + 2\nu_{yx}\right)\Big/\left(1 - \nu_{xy}\nu_{yx}\right)$.

AI2 = Invariant 2 from AC-2. $I_2 = G_{xy} - E_x\nu_{yx}\Big/\left(1 - \nu_{xy}\nu_{yx}\right)$.

Card D_1: Used if CNTL = 1.0.

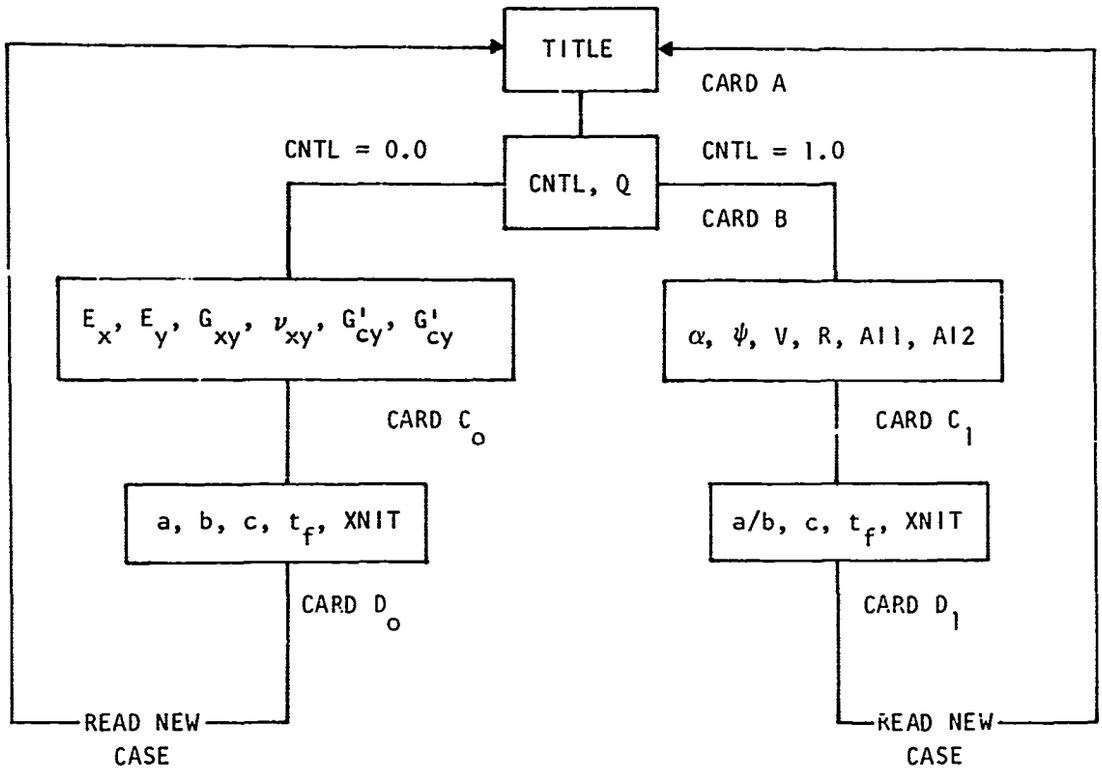
AØB = Panel aspect ratio a/b.

C = Core depth, c.

TF = Thickness of one face sheet, t_f.

XNIT = Maximum number of iterations to be allowed (usually set at 100).

The input data flow chart for AC-11 is:



Additional cases can be stacked one behind the other.

OUTPUT

The program output is of two types, depending on whether material property or material parameter input is desired. For material property input, the output includes the input data; the material parameters α, ψ, and V resulting from these data;. the number of iterations (NIT) carried out for $k_{s,even}$ and $k_{s,odd}$; and the final buckling load per inch, $N_{s,cr}$. For material parameter input, the input data are printed, as well as the two values of NIT mentioned previously, and the final buckling load as a function of $1/b^2$, since for this input no b was introduced.

```
C
C      PROGRAM AC11 ... H/C PANEL STABILITY FOR IN-PLANE SHEAR LOADING
C
C      THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
C           A = PANEL WIDTH ALONG X-DIRECTION
C           AI1 = INVARIANT 1 = EX*(1+EY/EX+2*MUYX)/(1-MUXY*MUYX)
C           AI2 = INVARIANT 2 = GXY - MUYX*EX/(1-MUXY*MUYX)
C           ALPHA = SQRT(EX/EY)
C           AOB = PANEL ASPECT RATIO A/B
C           B = PANEL LENGTH ALONG Y-DIRECTION
C           C = CORE DEPTH
C           CNTL = CONTROL CARD FOR DESIGNATING TYPE OF INPUT TO BE USED
C                  (=0 FOR MATL. PROPERTIES, OR =1 FOR MATL. PARAMETERS)
C           EIGMIN = EIGENVALUE ITERATION SOLUTION THAT TAKES THE SORT
C                    OF THE ABS VALUE OF SUCCEEDING EIGENVALUES
C           GCX = CORE SHEAR MODULUS IN THE X-DIRECTION
C           GCY = CORE SHEAR MODULUS IN THE Y-DIRECTION
C           PSI = SQRT(EX*EY)/(1-MUXY*MUYX)
C             O = NUMBER OF TERMS IN THE SERIES (MAXIMUM OF 13)
C           R = GCX/GCY
C           TF = THICKNESS OF ONE FACE SHEET
C           TITLE = ANY DESIRED ONE-LINE TITLE IN COLS.1-72
C           V = PI**2*PSI*TF*(C+TF)/(2*GCX*B**2)
C           XNIT = NUMBER OF ITERATIONS DESIRED IN THE SOLUTION
C
      DIMENSION  H( 85, 85), TITLE(18), BB(2)
   10 WRITE(6,50)
   50 FORMAT(1H1)
      READ(5,100) TITLE
  100 FORMAT(18A4)
      WRITE(6,100) TITLE
C.....READ MATERIAL PROPERTIES
      READ(5,110) CNTL,O
  110 FORMAT(6F12.0)
      IF(CNTL) 101,109,101
C.....READ PARAMETRIC PROPETIES
  101 READ(5,110)  ALPHA,PSI,V,R,AI1,AI2,AOB,C,TF,XNIT
      CC1=AI1/PSI-ALPHA-1.0/ALPHA
      BETA=1.5*CC1+2.*AI2/PSI
      GAMA=0.5*CC1+AI2/PSI
      UMXY=ALPHA*(BETA-2.0*GAMA)
      UMYX=UMXY/ALPHA**2
      AMDA=1.0-UMXY*UMYX
      EX=ALPHA*PSI*AMDA
      EY=EX/ALPHA**2
      GXY=AI2+UMYX*EX/AMDA
      SX=V*C/(C+TF)/AOB**2
      WRITE(6,115) ALPHA,PSI,V,R
  115 FORMAT(1H04X,28HTHE MATERIAL PARAMETERS ARE,//10X,6HALPHA=F6.3,5X,
     14HPSI=F12.5,5X,2HV=F7.4,5X,8HGCX/GCY=F6.3)
      WRITE(6,116) AOB,C,TF
  116 FORMAT(1H0,4X,28HTHE GEOMETERY PROPERTIES ARE,//12X,4HA/B=F7.3,6X,
     12HC=F7.4,10X,3HTF=F7.4)
      GO TO 131
C.....READ MATERIAL PROPETIES
  109 READ(5,110) EX,EY,GXY,UMXY,GCX,GCY
C.....READ PANEL GEOMETRY
  112 READ(5,110) A,B,C,TF,XNIT
      AOB=A/B
      UMYX=EY/EX*UMXY
      ALPHA=(EX/EY)**.5
      AMDA=1.0-UMYX*UMXY
```

```
      PSI=(FX*FY)**.5/AMDA
      GAMA=GXY/PSI
      BETA=ALPHA*UMYX+2.0*GAMA
      P=C+TF
      T=PSI*TF
      D=T*P**2/2.0
      SX=C*D**3.8096/(P*A)**2/GCX
      R=GCX/GCY
      V=SX*P*AOB**2/C
C.....WRITE INPUT DATA
  118 WRITE(6,120) FX,GCX,FY,GCY,GXY,UMXY,UMYX
  120 FORMAT(1HO,4X,28HTHE MATERIAL PROPERTIES ARE //10X,15HFACE SHEET E
     1X =E13.5,10X, 5HSHEAR GX =E13.5/21X,4HEY =E13.5,15X,4HGY =E13.5/20X
     25HGXY =E13.5/19X,5HMUXY =F8.4/19X,6HMUYX =F8.4)
C
      WRITE(6,130) A,B,C,TF,AOB
  130 FORMAT(1HO,4X,22HTHE PANEL GEOMETRY IS //10X,16HPANEL LENGTH A =F8
     1.3,5X,15HPANEL WIDTH B =F8.3/12X,14HCORE DEPTH C =F8.3, 5X,25HFACE
     2 SHEET THICKNESS TF =F8.3/21X,5HA/B =F8.3)
      WRITE(6,125) ALPHA,PSI,V,R
  125 FORMAT(1HO4X,28HTHE MATERIAL PARAMETERS ARE,//10X,6HALPHA=F6.3,5X,
     14HPSI=E12.5,5X,2HV=F7.4,5X,8HGCX/GCY=F6.3)
C.....CALCULATE CONSTANTS
  131 SY=R*SX
      P=C+TF
      T=PSI*TF
      D=T*P**2/2.0
      AOB2=AOB**2
      AOB4=AOB2**2
      IO=IO+.001
C.....XLMN CONSTANTS
      XLMN1=GAMA*AOB2
      XLMN2= R  /ALPHA*AOB2
      XLMN3=  R *GAMA
C.....XKMN CONSTANTS
      XKMN1=2.0*BETA*AOB2
      XKMN2=AOB4/ALPHA
C.....FMN CONSTANT
      FMN1=1.0-BETA**2
C.....VMN CONSTANTS
      XVMN1=AOB2*SX
      XVMN2=SX*SY
C.....DF CONSTANT
      DF=TF**3/12.0*PSI
      DFOD=DF/D
C.....CALCULATE VARIABLE CONSTANTS AFTER CHECKING (M+N)
      IP=1
  134 I=0
      NIT=XNIT
  140 DO 200 M=1,IO
      A1=M
      HA=4.24227*A1
      A2=A1**2
      A4=A2**2
      IF(M-2*(M/2)) 145,150,145
  145 NS=IP
      GO TO 155
  150 NS=3-IP
  155 DO 200 N=NS,IO,2
      I=I+1
      B1=N
```

```
      HR=HA*B1
      B2=B1**2
      B4=B2**2
C.....CALCULATE VARIABLE CONSTANTS
      XLMN   =ALPHA*A2+XLMN1*B2+XLMN2*B2+XLMN3*A2
      XKMN   =ALPHA*A4+XKMN1*A2*B2+XKMN2*B4
      FMN    =  XFMN1+GAMA*XKMN
      VMN  =(XKMN+(XVMN1*B2+SY*A2)*FMN)/(1.0+XLMN*SX+XVMN2*FMN)
      VMNI   =DFOO*XKMN
      AA=(2.0*VMNI+VMN)/AOB
C.....CHECK TO SEE IF (M+R) AND (N+S) ARE ODD OR EVEN
      J=0
      DO 190 IR=1,IQ
      RI=IR
      HR=HB*RI
      RI2=RI**2
      IF(IR-2*(IR/2)) 160,170,160
  160 ISS=IP
      GO TO 180
  170 ISS=3-IP
  180 DO 190 IS=ISS,IQ,2
      SI=IS
      HC=HR*SI
      SI2=SI**2
      J=J+1
      IF(M+IR-2*((M+IR)/2)) 182,187,182
  182 IF(N+IS-2*((N+IS)/2)) 185,187,185
  185 H(I,J)=-HC    /(A2-RI2)/(B2-SI2)/AA
      GO TO 190
  187 H(I,J)=0.0
  190 CONTINUE
  200 CONTINUE
      IQ2=(IQ**2+2-IP)/2
      DELTA=.001
      CALL EIGMIN(IQ2,DELTA,H,BB(IP),NIT)
      WRITE(6,217) NIT
  217 FORMAT(1HO,5X,4HNIT=I6/)
      IF(IP-2)220,240,240
  220 IP=2
      GO TO 135
C.....PRINT OUT EVEN AND ODD KS  WHICH ARE BB(1) AND BB(2)
  240 BB(1)=BB(1)/AOB2
      BB(2)=BB(2)/AOB2
      IF(BB(1)-BB(2)) 245,250,250
  245 XKS=BB(1)
      GO TO 255
  250 XKS=BB(2)
  255 IF(CNTL) 256,260,256
  256 XKNXCR=XKS*9.8696*D
      WRITE(6,258) XKNXCR
  258 FORMAT('0'5X'NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) ='
     1E13.5'/B**2')
      GO TO 280
  260 XNCRS=XKS*9.8696*D/B**2
      WRITE(6,265) IQ,BB(1),BB(2)
  265 FORMAT(1HO,38H****THE VALUES FOR KS  WHEN THERE ARE I3,22H CMN COE
     1FFICIENTS ARE,/20X,10H KS EVEN =E13.5,10X, 9H KS ODD =E13.5)
      WRITE(6,270) XNCRS
  270 FORMAT('0'5X'NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * H**2) ='
     1E13.5)
  280 GO TO 10
      END
```

```
      SUBROUTINE EIGMIN(NN,DELTA,BINV,ALAMIN,NIT)
      DIMENSION C( 85),B1( 85),BINV( 85,85)
C
      II=1
    1 DO 2 I=1,NN
      C(I)=1.0
    2 CONTINUE
      N=1
    3 DO 5 I=1,NN
      B1(I)=0.0
    4 DO 5 J=1,NN
      B1(I)=B1(I)+BINV(I,J)*C(J)
    5 CONTINUE
      C1=ABS(B1(1))
      ALAMDA=C1
    8 DO 9 I=1,NN
      C(I)=B1(I)/C1
    9 CONTINUE
      IF(N-1) 11,10,11
   10 N=N+1
      BLAMDA=ALAMDA
      C4=(BLAMDA**LAMDA)**.5
      GO TO 3
   11 C2=(ALAMDA*BLAMDA)**.5
      C3=ABS(C2-C4)
      IF(C3-DELTA)  14,14,12
   12 BLAMDA=ALAMDA
      C4=C2
      II=II+1
      IF(II-NIT) 13,14,14
   13 GO TO 3
   14 NIT=II
      ALAMIN=1.0/
      RETURN
      END
```

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T L E	Card A	Title = Any desired one card title - COL. 1-72
13			
25			
37			
49		73 80	
61			
1	C N T L	Card B	CNTL = Input data form indicator {0.0 for matl. prop. / 1.0 for matl. para.}
13			
25	Q		Q = Range of M and N (Maximum of 13)
37			
49		73 80	
61			
1	E X	IF CNTL = 0.0 Card C_o	EX = Young's modulus in X-direction
13	E Y		EY = Young's modulus in Y-direction
25	G X Y		GXY = Shear
37	U M X Y		UMXY = Poissons ratio ν_{xy}
49	G C X	73 80	GCX = Core shear modulus in X-direction - G'_{cx}
61	G C Y		GCY = Core shear modulus in Y-direction - G'_{cy}
1	A	IF CNTL = 0.0 Card D_o	A = Length of panel in X-direction
13	B		B = Width of panel in Y-direction
25	C		C = Core depth
37	T F		TF = One face sheet thickness
49	X N I T	73 5	XNIT = Max. no. of iterations
61			

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A L P H A	IF CNTL = 1.0 CARD C_1	ALPHA = $\sqrt{E_x/E_y}$
13	P S I		PSI = $\sqrt{E_x E_y} / (1 - \nu_{xy}\,\nu_{yx})$
25	V		V = $\pi^2 \Psi t_f (c + t_f) / (2\,G'_{cx}\,b^2)$
37	R		R = G'_{cx}/G'_{cy}
49	A I 1	73 80	AI1 = $E_x (1 + E_y/E_x + 2\,\nu_{yx}) / (1 - \nu_{xy}\,\nu_{yx})$
61	A I 2		AI2 = $G_{xy} - E_x \nu_{yx}/(1 - \nu_{xy}\,\nu_{yx})$
1	A Ø B	IF CNTL = 1.0 CARD D_1	AØB = Panel aspect ratio a/b
13	C		C = Core depth
25	T F		TF = One face sheet thickness
37	X N I T		XNIT = Max No. of iterations
49		73 80	
61			
1			
13			
25			
37			
49		73 80	
61			
1			
13			
25			
37			
49		73 80	
61			

EXAMPLE PROBLEM

Consider a honeycomb panel to be made of $[0/\pm45/0]_S$ face sheets with a core depth of 1/2 inch, G'_{cx} = 126,000 psi, G'_{cx}/G'_{cy} = 2.0, and an overall panel size of 15.0 x 15.0 inches. To illustrate the use of the program, both types of input will be used. The material parameters and material properties are taken from the AC-2 and AC-5 sample problems.

The data sheets are shown for both types of input near the end of this section. Note that for the material parameter input,

$$N_{s,cr} = 5.1021 \times 10^6 / b^2$$

and if b = 15, as in this example,

$$N_{s,cr} = 5.1021 \times 10^6 / (15)^2 = 22676 \text{ lb/in.}$$

as found from the cutput.

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0/+-45/0)S		Title
13	LAMINATE AT		
25	RØØM TEMPERA		
37	TURE		
49		73 9	
61		10	
1	0.0		CNTL
13	13.0		Q
25			
37			
49		73 8	
61		2.0	
1	16670000.		EX
13	5058000.		EY
25	43000000.		GXY
37	.7015		UMXY
49	1260000.	73 9	GCX
61	630000.	30	GCY
1	150.0		A
13	150.0		B
25	0.5		C
37	0.0416		TF
49	100.	73 =	XNIT
61		40	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0/+-45/0)S		TITLE
13	LAMINATE AT		
25	RØØM TEMPERA		
37	TURE		
49		73 80	
61		5.0	
1	1.0		CNTL
13	13.0		Q
25			
37			
49		73 80	
61		6.0	
1	1.815		ALPHA
13	10794000.		PSI
25	.04233		V
37	2.0		R
49	33884000.	73 80	AI1
61	128620	70	AI2
1	1.0		AØB
13	0.5		C
25	.0416		TF
37	100.		XNIT
49		73 80	
61		8.0	

```
(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

     THE MATERIAL PROPERTIES ARE

          FACE SHEET EX =  0.16670E 08          CORE GX =  0.12600E 06
                     EY =  0.50580E 07               GY =  0.63000E 05
                    GXY =  0.43000E 07
                   MUXY =  0.7015
                   MUYX =  0.2128

     THE PANEL GEOMETRY IS

          PANEL LENGTH A =  15.000      PANEL WIDTH B =  15.000
            CORE DEPTH C =   0.500      FACE SHEET THICKNESS TF =   0.042
                     A/B =   1.000

     THE MATERIAL PARAMETERS ARE,

          ALPHA= 1.815      PSI= 0.10794E 08      V= 0.0423      GCX/GCY= 2.000

      NIT=      7

      NIT=     11

****THE VALUES FOR KS  WHEN THERE ARE  13 CMN COEFFICIENTS ARE,
                    KS EVEN = 0.78484E 01          KS ODD = 0.84673E 01

      NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) =  0.22670E 05

0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

     THE MATERIAL PARAMETERS ARE,

          ALPHA= 1.815      PSI= 0.10794E 08      V= 0.0423      GCX/GCY= 2.000

     THE GEOMETERY PROPERTIES ARE

           A/B=  1.000       C= 0.5000          TF= 0.0416

     NIT=      7

     NIT=     11

     NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) =  0.51021E 07/B**2
```

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COMPUTER PROGRAM AC-12

HAT- AND Z-STIFFENED PANEL STABILITY AND STIFFNESS

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-12

ABSTRACT

Computer program AC-12 is designed to provide the optimum minimum length and the resulting face sheet and stringer stresses for a hat- or Z-stiffened panel. These quantities are given as a function of load level and stiffener spacing as well as other geometrical and material properties. The loading is uniaxial in the x-direction.

The optimum minimum lengths of the panels are found by equating the local and Euler buckling equations and solving for the length. A program listing is given following the text in this section.

INPUT DATA

The input data, as shown on the sample data sheets in this section, are given more completely in the following listing:

Card A:

BMAT = Any desired one-card alphanumeric title limited to columns 1-72.

Card B: Face-sheet properties

A11 = $E_x t/\left(1-\nu_{xy}\nu_{yx}\right)$.

A12 = $\nu_{yx}\, A_{11}$.

A16 = Defined by equation 9 of this volume and is 0.0 for symmetric laminates.

A22 = $E_y t/\left(1-\nu_{xy}\nu_{yx}\right) = \left(E_y/E_x\right) A_{11}$.

A26 = Defined by equation 9 of this volume and is 0.0 for symmetric laminates.

A66 = $G_{xy}\, t$

Card C:

D11 = Flexural rigidity of the sheet in the x-direction (load direction).

AEXF = E_x^f = Face-sheet modulus in the x-direction.

T = Face-sheet thickness.

TP = Indicator for Z (TP = 0.0) and hat stiffeners.

AK7 = Buckling coefficient from equation 131 of this volume.

AKS = Stiffener buckling coefficient found from equation 132 of this volume.

Card D:

AKF = Face-sheet buckling coefficient defined by equation 132 of this volume.

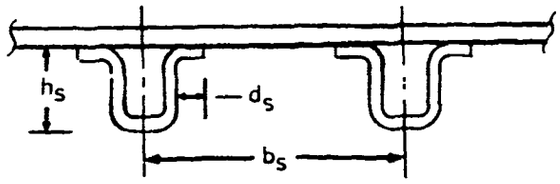
Card E:

AEXS = Young's modulus of the stiffener in the x-direction.

TNBS = Total number of stiffener spacing to be considered.

TNNX = Total number of N_x loadings to be considered.

DSØHS = Hat geometric parameter d_s/h_s (0.0 for Z-stiffener, i.e., when TP = 0.0), as defined by the following sketch:



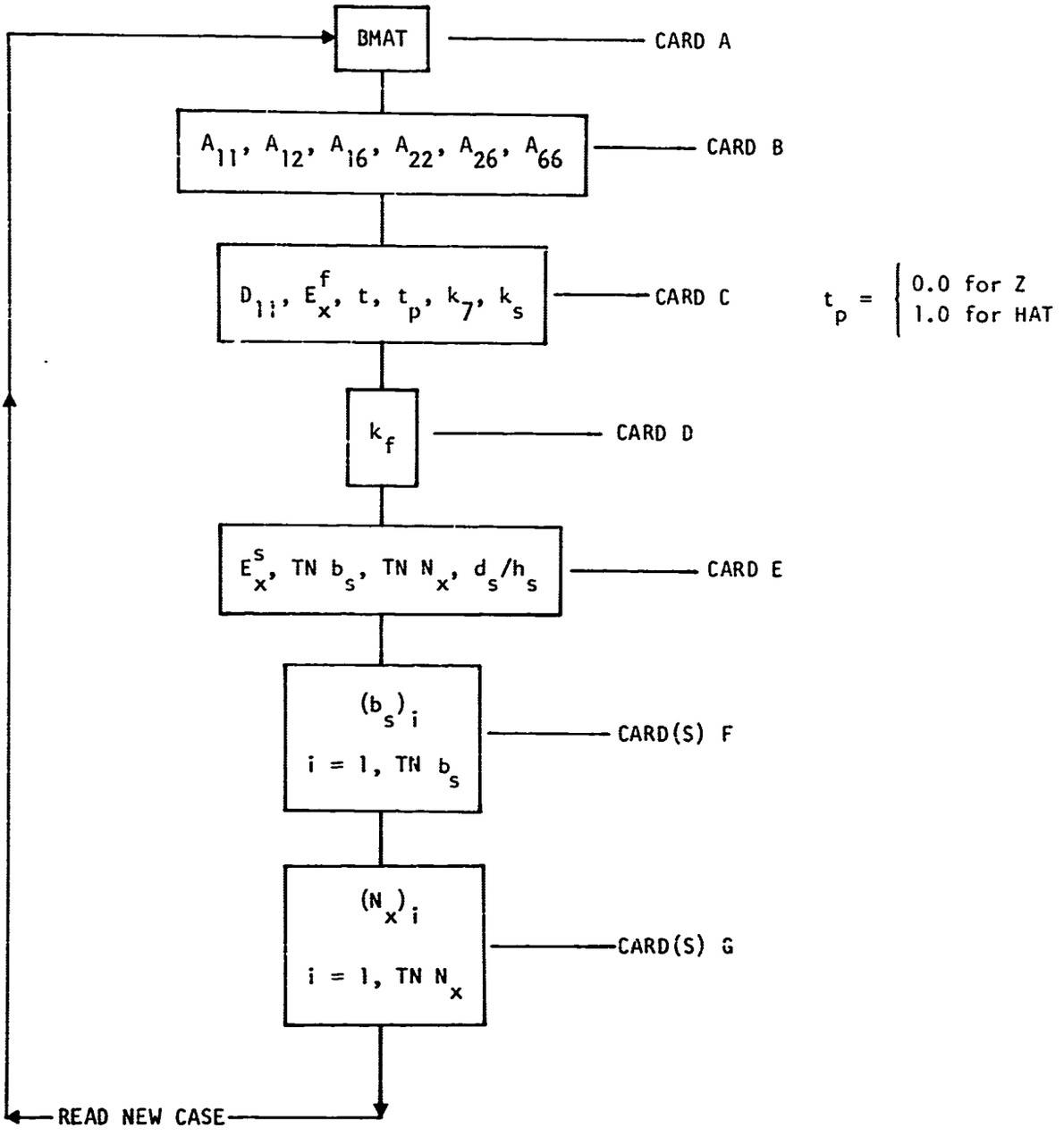
Card(s) F:

XBS(I) = Stiffener spacings $(b_s)_i$ (maximum of 30).

Card(s) G:

BNX(I) = Load levels to be considered, N_{x_i} (maximum of 30).

The input data flow chart for AC-12 is:



```
C
C     AC12...Z- AND HAT-STIFFENED PANEL STABILITY AND STIFFNESS
C
C     THE INPUT DATA IS AS FOLLOWS.....
C           BMAT = ANY DESIRED ONE-CARD TITLE IN COLS. 1-72
C           A11 = EX*T/(1-MUXY*MUYX)
C           A12 = MUYX*A11
C           A16 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
C           A22 = EY*T/(1-MUXY*MUYX) = A11*EY/EX
C           A26 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
C           A66 = GXY*T
C           T = LAMINATE THICKNESS
C           D11 = SHEET FLEXURAL STIFFNESS IN THE X-DIRECTION FROM
C                 PROGRAM AC2 OR EQUATION 9
C           AEXF = FACE SHEET YOUNG'S MODULUS IN THE X-DIRECTION
C           TP = INDICATOR FOR Z(TP=0.0) AND HAT(TP=1.0)
C           AK7 = BUCKLING COEFFICIENT FOUND FROM EQUATION 131
C           AKS = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
C           AKF = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
C           AEXS = YOUNG'S MODULUS EX OF THE STIFFENER
C           TNBS = TOTAL NUMBER OF STIFFENER SPACINGS
C           TNNX = TOTAL NUMBER OF LOADINGS TO BE CONSIDERED
C           DSOHS = HAT STIFFENER GEOMETRY RATIO
C           BNX(I) = LOADINGS
C           XBS(I) = STIFFENER SPACINGS
C
      DIMENSION  BMAT(18),BNX(30),XBS(30)
    1 READ(5,2) (BMAT(M),M=1,18)
    2 FORMAT(18A4)
   45 WRITE(6,46)(BMAT(M),M=1,18)
   46 FORMAT(1H1,18A4)
    4 READ(5,3)A11,A12,A16,A22,A26,A66,D11,AEXF,T,TP,AK7,AKS,AKF
    3 FORMAT(6F12.0)
C
C     READ IN  STIFFENER PROPERTIES
C
    5 READ(5,3) AEXS,TNBS,TNNX,DSOHS
    7 IA=TNBS
      IB=TNNX
      READ(5,3)(XBS(I),I=1,IA)
      READ(5,3)(BNX(I),I=1,IB)
      DO 200 J=1,IB
   50 WRITE(6,51)BNX(J),T
   51 FORMAT(24H0            NX(LB/INCH)=F7.1,7H      T=F7.4)
   40 WRITE(6,41)
   41 FORMAT(99H0  BS        HS        TS         FS            D1        L,C
     1RIT.       SIGS       SIGF                 DS                        )
      DO 200 I=1,IA
       ANX=BNX(J)
        BS=XBS(I)
      BETA= (AK7/AKS)**.5
      ALFA =(AEXS*AKF/(AEXF*AKS))**.5
      IF(TP)17,16,17
C
C     LOCAL BUCKLING
C
C     CALCULATION OF GEOMETRY OF Z-STIFFENER ELEMENTS TO INSURE
C     SIMULTANEOUS LOCAL BUCKLING
```

```
   16 HS =(( ANX*AEXS/(AKS*(ALFA*T/BS)**2) -AEXF*T)*BS**2/(ALFA*T *AEXS
     1*(2.*BETA +1.)) )**.5
      TS=(ALFA * HS * T)/BS
      FS= BETA *HS
      GO TO 20
C
C     CALCULATION OF GEOMETRY OF HAT-STIFFENED ELEMENTS TO INSURE
C     SIMULTANEOUS LOCAL BUCKLING
C
   17 HS =(( ANX*AEXS/(AKS*(ALFA*T/BS)**2) -AEXF*T)*BS**2/(ALFA*T *AEXS
     1*2.*(1.5+DSOHS)))**.5
      DS= DSOHS*HS
      TS= ALFA*HS *T/BS
      FS=.5* HS
   20  IF(TP)9,8,9
    8 AK1=AEXS*TS*(2.*FS+HS)
      AK2=AEXS*TS*(FS*(HS+.5*(T-TS))+.5*HS*(HS+T)+FS*.5*(TS+T))
      C1=-AEXS*TS*(FS*(HS+.5*(T-TS))+.5*HS*(HS+T)+FS*.5*(TS+T))
      C2=-AEXS*TS*(FS*(HS*(HS + T -TS) +TS*(TS/3.-T/2.)+T**2/4.) +FS*(T*
     1*2/4. +T*TS/2.+TS**2/3.) +HS*(HS**2/3.+HS*T/2. +T**2/4.))
      GO TO 10
    9 AK1= 2.*AEXS*TS*(DS+HS+FS)
      AK2=AEXS*TS*(DS*(TS+T)+HS*(HS+T) + 2.*FS * (HS +T/2. -TS/2.))
      C1=- TS*AEXS *( DS*(TS+T) +HS*(HS+T) +2.*FS*(HS +T/2. -TS/2.))
      C2= -AEXS *2.*TS *( FS*(HS*(HS+T-TS)  +TS*(TS/3. -T/2.)+T**2/4. )
     1 +DS*(TS**2/3. +T*TS/2. +T**2/4.)+HS*(HS**2/3. +HS*T/2. +T**2/4.))
   10 DEL=(AK2/BS) /( (A12-A16*A26/A66)**2 -(AK1/BS  +A11 -A16**2/A66) *
     1(A22-A26**2/A66))
      BETA= DEL*(A22-A26**2/A66)
      D1= -(C1/BS)*BETA-C2/BS + D11
      ANXL2 = 9.8696*D1
      XLCR=(ANXL2/ANY)**.5
C
C     DETERMINATION OF STIFFENER AND SHEET PROPERTIES
C
      IF(TP)14,12,14
   12 C3= AEXF*T + AEXS*TS*(2.*FS+ HS -2.*TS)/BS
      GO  TO  15
   14 C3= AEXF*T +  AEXS*TS*2.*(DS+HS+  FS)/BS
   15 SIGF= ANX*AEXF/C3
      SIGS= ANX*AEXS/C3
  100 WRITE(6,101)BS,HS,TS,FS,D1,XLCR ,SIGS,SIGF,DS
  101 FORMAT(F6.3,3F9.3,F14.2,F14.1,2F11.1,F16.3)
  200 CONTINUE
      GO TO 1
      END
```

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	B M A T	A	BMAT = ANY ONE-LINE TITLE IN COLS. 1-72
13			
25			
37			
49		73 80	
61			
1	A 1 1	B	A11 = $E_x t/(1 - \nu_{xy}\nu_{yx})$
13	A 1 2		A12 = ν_{yx} A11
25	A 1 6		A16 = Defined by Equ. 9 (0.0 for Symmetri nates)
37	A 2 2		A22 = $E_y t/(1 - \nu_{xy}\nu_{yx})$
49	A 2 6	73 80	A26 = Defined by Equ. 9 (0.0 for Symmetric Laminates)
61	A 6 6		A66 = G_{xy} t
1	D 1 1	C	D11 = Flexural rigidity of the sheet in the X-direction
13	A E X F		E_x^f = Face sheet modulus in the X-direction
25	T		t = Face sheet thickness
37	T P		TP = Indicator for Z (TP = 0.3) and Hat (TP = 1.0)
49	A K 7	73 80	K_7 = Buckling coefficient found from Equ. 131
61	A K S		K_s = Stiffener buckling coefficient found from Equ. 132.
1	A K F	D	K_f = Face sheet buckling coefficient found from Equ. 132
13			
25			
37			
49		73	
61			

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	A E X S	E	AEXS = Young's modulus of stiffener in the X-direction
13	T N B S		TNBS = Total number of stiffener spacings to be considered
25	T N N X		TNNX = Total number of loadings N_x to be considered
37	D S Ø H S		DSØHS= Hat geometric parameter (0.0 for Z - stiffener)
49		73 80	
61			
1	X B S (1)	F	XBS(1) = First stiffener spacing
13	X B S (2)		XBS(2) = Second stiffener spacing
25	X B S (3)		XBS(3) = Third stiffener spacing
37			
49		73 80	
61			(maximum of 30)
1	B N X (1)	G	BNX(1) = First applied load level
13	B N X (2)		BNX(2) = Second applied load level
25	B N X (3)		BNX(3) = Third applied load level
37			
49		73 80	
61			(Maximum of 30)
1			
13			
25			
37			
49		73	
61			

EXAMPLE PROBLEM

An example problem is given for a hat-stiffened and a Z-stiffened $[0/\pm45/0]_S$ face sheet. The input values are given on the data sheets at the end of this section. For this laminate, from AC-2, we take the face-sheet properties as:

$$E_x = 16.67 \times 10^6 \text{ psi}$$

$$E_y = 5.06 \times 10^6 \text{ psi}$$

$$G_{xy} = 4.30 \times 10^6 \text{ psi}$$

$$\nu_{xy} = 0.7015$$

$$\nu_{yx} = \left(E_y/E_x\right)\nu_{xy} = 0.2129$$

The laminate thickness is 8 plies x 0.0052 inch/ply = 0.0416 inch.

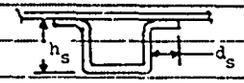
The buckling coefficients k_7, k_s, k_f, are found from equations 131 and 132 of this volume.

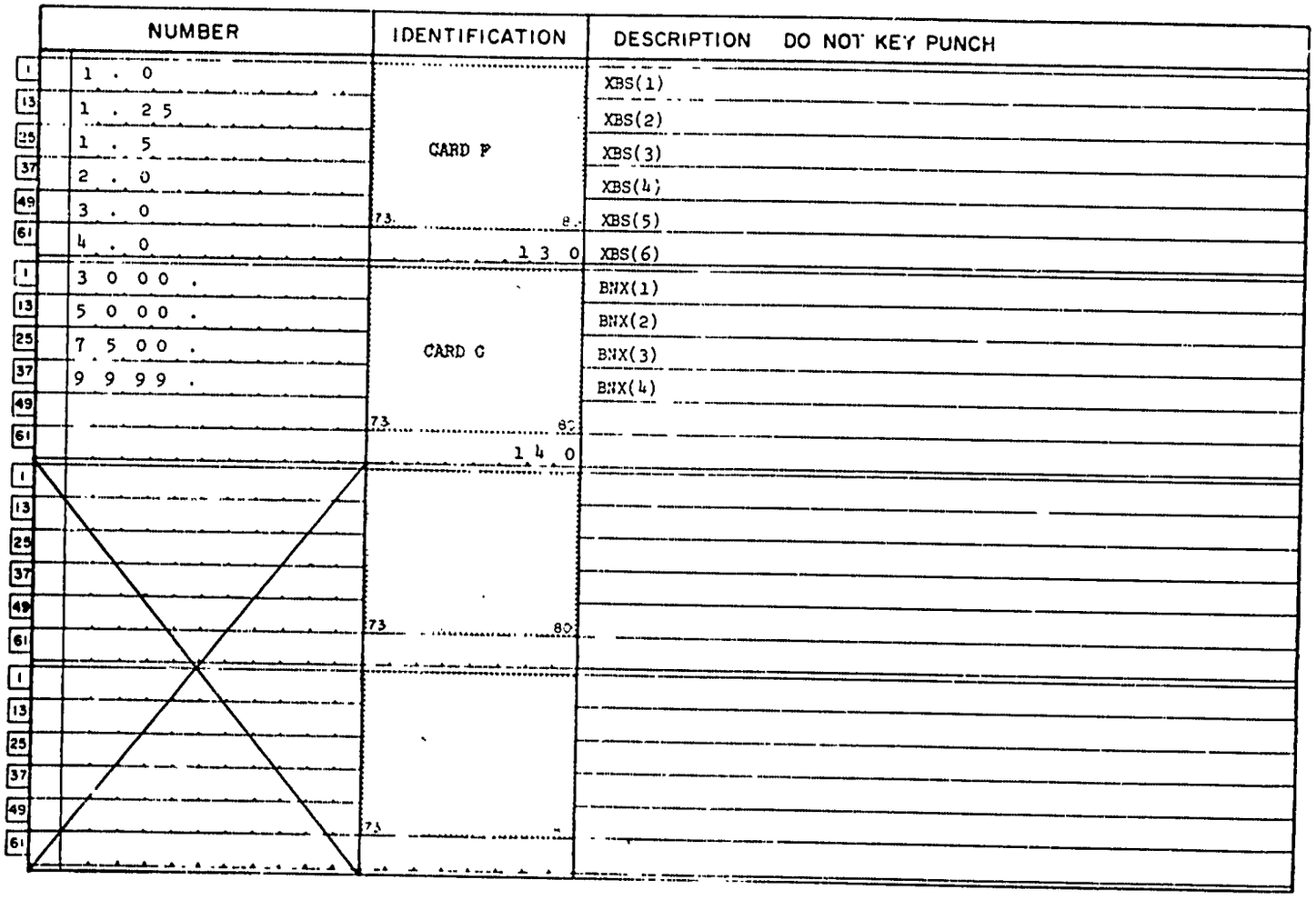
The results are given first for the Z-stiffened panels and then for the hat-stiffened panels. The panel critical lengths may be read directly from these results.

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	Z E E- S T I F F E N E	CARD A	
13	D P A N E L S W I T		
25	H (0 / + - 4 5 / 0)		
37	S F A C E S A T R		
49	Ø Ø M T E M P E R A T	73 80	
61	U R E	1 0	
1	8 0 9 0 0 0	CARD B	A11
13	1 7 0 0 0 0		A12
25	0 . 0		A16
37	2 4 5 0 0 0 .		A22
49	0 . 0	73 80	A26
61	1 7 7 0 0 0 .	2 0	A66
1	1 2 9 . 3	CARD ?	D11
13	1 6 6 7 0 0 0 0 .		AEXF
25	0 . 0 4 1 6		T
37	0 . 0		TP
49	6 9 7 0 0 0	73 80	
61	1 8 0 0 0 0 0 0 0 .	3.0	AKS
1	3 8 0 0 0 0 C 0 0 .	CARD D	AKF
13			
25			
37			
49		73 80	
61		4.0	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	2.09000.00.	CARD E	AEXS
13	6.0		TNBS
25	4.0		TNNX
37			
49		73 80	
61		50	
1	1.0	CARD F	XBS(1)
13	1.25		XBS(2)
25	1.50		XBS(3)
37	2.0		XBS(4)
49	3.0	73 80	XBS(5)
61	4.0	60	XBS(6)
1	3000	CARD G	BNX(1)
13	5000		BNX(2)
25	7500		BNX(3)
37	9999		BNX(4)
49		73 80	
61		70	
1 H	AT-STIFFENE	CARD A	
13 D	PANELS WIT		
25 H	(0/+-45/0)		
37 S	FACES AT R		
49 Ø	ØM TEMPERAT	73 80	
61 U	RE	80	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	8 0 9 0 0 0	CARD B	A11
13	1 7 0 0 0 0		A12
25	0 . 0		A16
37	2 4 5 0 0 0		A22
49	0 . 0	73	A26
61	1 7 7 0 0 0 .	90	A66
1	1 2 9 . 3	CARD C	D11
13	1 6 6 7 0 0 0 0 .		AEXF
25	0 . 0 4 1 6		T
37	1 . 0		TP
49	6 9 7 0 0 0 .	73	AK7
61	1 8 0 0 0 0 0 0 .	100	AKS
1	3 8 0 0 0 0 0 0 .	CARD D	AKF
13			
25			
37			
49		73	
61		110	
1	2 9 9 0 0 0 0 0	CARD E	AEXS
13	6 . 0		TNBS
25	4 . 0		TNNX
37	2 . 1 2 1		DSØHS
49		73	
61		120	





	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1.0	CARD P	XBS(1)
13	1.25		XBS(2)
25	1.5		XBS(3)
37	2.0		XBS(4)
49	3.0		XBS(5)
61	4.0	73 80 130	XBS(6)
1	3000.	CARD G	BNX(1)
13	5000.		BNX(2)
25	7500.		BNX(3)
37	9999.		BNX(4)
49			
61		73 80 140	
1			
13			
25			
37			
49			
61		73 80	
1			
13			
25			
37			
49			
61		73 80	

ZEE-STIFFENED PANELS WITH (0/+-45/0)S FACES AT ROOM TEMPERATURE

NX(LB/INCH)= 3000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.141	0.011	0.028	797.37	1.6	119171.8	66441.3	0.0
1.250	0.479	0.031	0.094	33442.80	10.5	78528.2	43781.5	0.0
1.500	0.824	0.044	0.162	162204.69	23.1	54954.5	30638.5	0.0
2.000	1.669	0.068	0.328	1207161.00	63.0	30872.5	17212.2	0.0
3.000	4.051	0. 09	0.797	15387825.00	225.0	13578.3	7570.3	0.0
4.000	7.378	0.169	1.452	88648192.00	540.0	7579.6	4225.8	0.0

NX(LB/INCH)= 5000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.413	0.033	0.081	27976.11	7.6	124502.4	69413.3	0.0
1.250	0.772	0.050	0.152	167891.00	18.2	80341.7	44792.5	0.0
1.500	1.200	0.065	0.236	584217.25	34.0	55687.5	31047.2	0.0
2.000	2.278	0.092	0.448	3638542.00	84.7	31044.7	17308.2	0.0
3.000	5.347	0.144	1.052	43504768.00	293.0	13600.5	7582.6	0.0
4.000	9.639	0.195	1.897	247798400.00	699.4	7584.8	4228.7	0.0

NX(LB/INCH)= 7500.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.598	0.048	0.118	96869.94	11.3	127350.7	71001.2	0.0
1.250	1.027	0.067	0.202	443716.69	24.2	81280.0	45315.7	0.0
1.500	1.546	0.083	0.304	1419840.00	43.2	56061.3	31255.6	0.0
2.000	2.863	0.116	0.563	8396794.00	106.1	31131.5	17356.6	0.0
3.000	6.619	0.179	1.302	98444784.00	359.9	13611.5	7586.7	0.0
4.000	11.875	0.240	2.337	558737920.00	857.5	7587.4	4230.1	0.0

NX(LB/INCH)= 9999.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.739	0.060	0.145	196830.38	13.9	128823.7	71822.4	0.0
1.250	1.230	0.080	0.242	832252.25	28.7	81757.4	45581.8	0.0
1.500	1.828	0.099	0.360	2590814.00	50.6	56250.1	31360.8	0.0
2.000	3.367	0.135	0.659	15056129.00	121.9	31175.1	17380.9	0.0
3.000	7.682	0.207	1.512	175337600.00	416.0	13617.1	7591.9	0.0
4.000	13.751	0.278	2.706	993924864.00	990.5	7588.6	4230.9	0.0

HAS-STIFFENED PANELS WITH (Z+-45/0)S FACES AT FOUR TEMPERATURES

DX(LB/IN)= 3000.0 T= 0.0417

RS	BS	TS	ES	DJ	L,CRIT.	SIGS	SIGF	DS
1.000	0.062	0.005	0.031	256.69	0.9	117951.9	65761.1	0.131
1.250	0.210	0.016	0.105	5066.02	4.1	75689.1	42087.1	0.469
1.500	0.361	0.020	0.181	25206.95	9.1	52623.1	29227.2	0.767
2.000	0.732	0.030	0.366	208995.19	26.2	29488.0	16440.3	1.553
3.000	1.777	0.068	0.889	3024102.00	99.7	13105.8	7306.8	3.769
4.000	3.236	0.065	1.619	18427360.00	266.2	7372.0	4110.1	6.964

DX(LB/IN)= 5000.0 T= 0.0416

RS	BS	TS	ES	DJ	L,CRIT.	SIGS	SIGF	DS
1.000	0.111	0.015	0.090	4366.29	2.9	117952.0	65761.2	0.384
1.250	0.339	0.022	0.169	26855.86	7.3	75689.3	42087.2	0.719
1.500	0.526	0.028	0.263	99606.00	14.0	52623.1	29227.2	1.116
2.000	0.999	0.040	0.500	623537.38	36.7	29488.0	16440.3	2.120
3.000	2.345	0.063	1.173	8963143.00	133.0	13105.8	7306.8	4.975
4.000	4.228	0.086	2.114	53025328.00	323.5	7372.0	4110.1	8.968

DX(LB/IN)= 7500.0 T= 0.0416

RS	BS	TS	ES	DJ	L,CRIT.	SIGS	SIGF	DS
1.000	0.262	0.021	0.131	15500.08	4.5	117951.9	65761.1	0.557
1.250	0.651	0.030	0.225	75008.75	10.0	75689.1	42087.1	0.995
1.500	0.678	0.037	0.339	254602.13	18.4	52623.2	29227.2	1.459
2.000	1.256	0.051	0.629	1655561.00	46.7	29488.0	16440.3	2.664
3.000	2.963	0.078	1.652	20807050.00	165.5	13105.8	7306.8	6.158
4.000	5.299	0.105	2.604	121382622.00	399.7	7372.0	4110.1	11.041

DX(LB/IN)= 9999.0 T= 0.0416

RS	BS	TS	ES	DJ	L,CRIT.	SIGS	SIGF	DS
1.000	0.324	0.026	0.162	32816.85	5.7	117951.9	65761.1	0.857
1.250	0.560	0.035	0.270	149162.06	12.1	75689.2	42087.1	1.149
1.500	0.802	0.043	0.401	490639.19	22.0	52623.1	29227.2	1.701
2.000	1.668	0.059	0.834	3047705.00	56.8	29488.0	16440.3	3.114
3.000	3.370	0.091	1.685	37565744.00	192.5	13105.8	7306.8	7.148
4.000	6.032	0.122	3.016	317596068.00	663.4	7372.0	4110.1	12.794

COMPUTER PROGRAM AC-40

N-PLY LAMINATE THERMOELASTIC ANALYSIS

ADVANCED FILAMENTARY COMPOSITE MECHANICAL PROPERTIES
COMPUTER PROGRAM DECK NO. AC-40

ABSTRACT

This program calculates the thermal stresses and coefficients of thermal expansion in both the individual ply axes and the laminate axes, given the macromechanical properties of each ply. The program is now set up to handle boron/epoxy laminates at room temperature (R.T.), 220° F, and 350° F. If other materials or temperatures are involved, the form of input may be altered accordingly.

PROGRAM INPUT DATA

The program input is minimal and consists of only two cards per case, which are shown on the sample input sheets following the text. The data are described as follows:

Card A:

N = Number of plies per set in the laminate.

M = Material temperature indicator. 1 = R.T., 2 = 220° F, and 3 = 350° F

DT = Temperature increment.

Card B:

TH(I) = Angle of ply I. There are six of these to a card (maximum of 20).

Note that only one "set" of a laminate need be coded in. For example, for a $[0/\pm 60/90_2]_{3S}$ laminate, only the 0°, +60°, -60°, 90°, and 90° plies need be input, giving a total N of five plies.

OUTPUT

The output consists of the DI, B, α, and $\bar{B}$ matrices; the coefficients of thermal expansion in the laminate axes $\bar{\alpha}_x$, $\bar{\alpha}_y$, and $\bar{\alpha}_{xy}$; and the thermal stresses in the laminate axes, σ_x, σ_y, and τ_{xy}, plus the thermal stresses in the individual ply natural axes, σ_L, σ_T, and τ_{LT}.

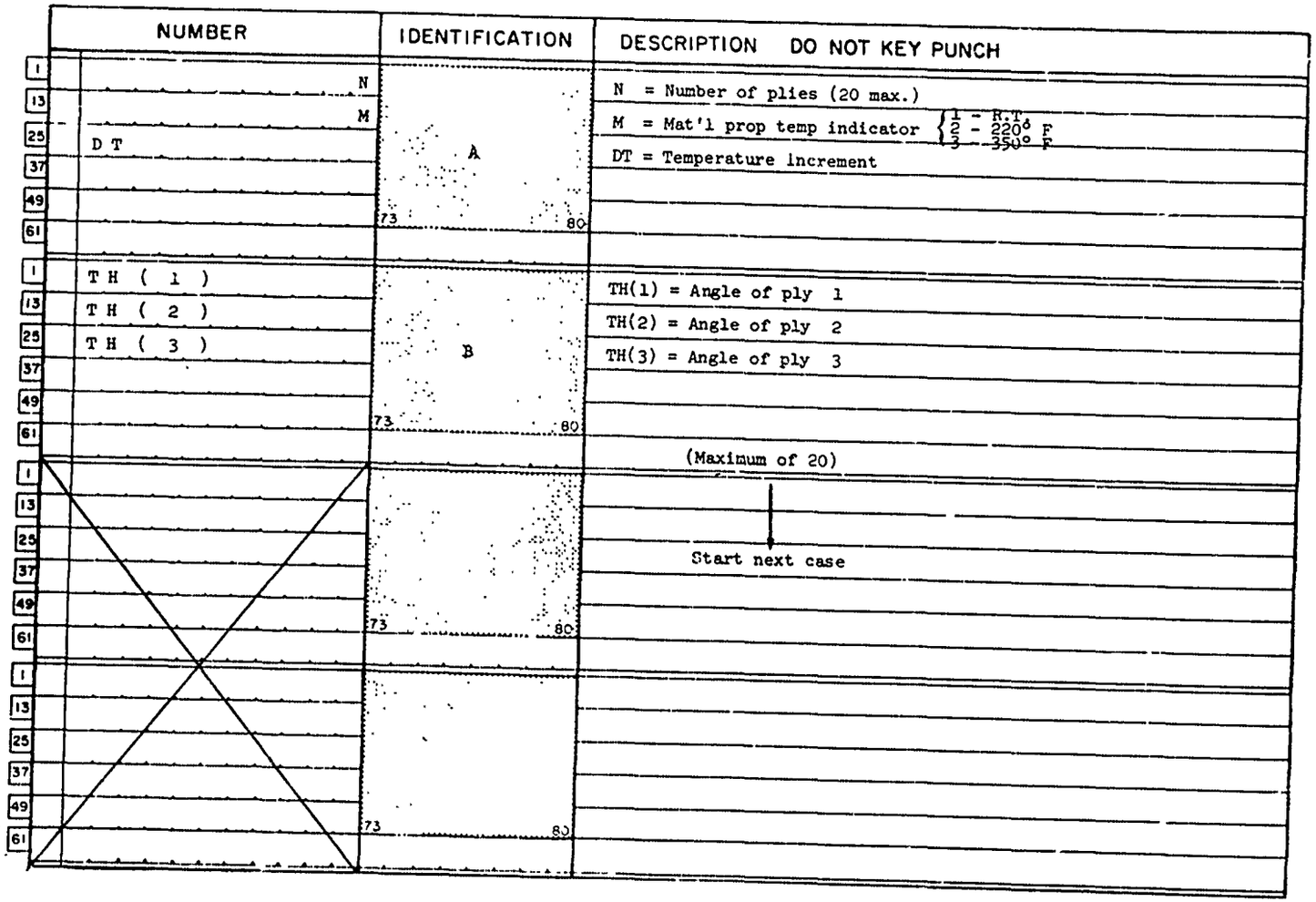
The output is shown on the sample output pages following the text.

```
C
C     PROGRAM AC40...N-PLY LAMINATE THERMOELASTIC ANALYSIS PROGRAM
C
C     INPUT DATA IS AS FOLLOWS.....
C           N = NUMBER OF PLIES
C           M = MATL. PROP. TEMP. INDICATOR....M=1 FOR R.T., M=2 FOR 220F,
C               M=3 FOR 350F
C           DT = TEMPERATURE INCREMENT IN DEGREES F
C           TH(I) = INDIVIDUAL PLY ANGLES THETA(I)
C
      DIMENSION EL(20),ET(20),GLT(20),ULT(20),UTL(20),AL(20),AT(20),
     1   H(20),A(20,3,3),SL(20),ST(20),SLT(20),TH(20)
      DIMENSION T(20,3,3),EM(20),EN(20),R(20,3,3),DI(20,3,3),AX(20),
     1   AXY(20),BB(3,3),SX(20),SY(20),SXY(20),BI(3,3),AY(20)
      DIMENSION D(20,3,3)
      EQUIVALENCE (D(1),DI(1))
    5 READ(5,10) N,M,DT
   10 FORMAT(2I12, F12.0)
C TH(I) = ANGLE BETWEEN NATURAL AXES OF I-TH PLY AND LAMINATE AXES
   15 READ(5,20) (TH(I),I=1,N)
   20 FORMAT(6F12.0)
C PROPERTIES OF INDIVIDUAL LAMINA
   40 DO 100 I=1,N
C CALCULATE ELEMENTS OF T MATRIX
      IF(M-2)42,43,44
C ROOM TEMPERATURE PROPERTIES
   42 EL(I)=29900000.
      ET(I)=2710000.
      GLT(I)=700000.
      ULT(I)=.21
      UTL(I)=ULT(I)*ET(I)/EL(I)
      AL(I)=.00000232
      AT(I)=.00001067
      H(I)=.0052
      GO TO 46
C 220 DEGREE PROPERTIES
   43 EL(I)=29900000.
      ET(I)=1420000.
      GLT(I)=377100.
      ULT(I)=.21
      UTL(I)=ULT(I)*ET(I)/EL(I)
      AL(I)=.000002414
      AT(I)=.00001394
      H(I)=.0052
      GO TO 46
C 350 DEGREE PROPERTIES
   44 EL(I)=29900000.
      ET(I)=1130000.
      GLT(I)=300000.
      ULT(I)=.21
      UTL(I)=ULT(I)*ET(I)/EL(I)
      AL(I)=.00000283
      AT(I)=.0000197R
      H(I)=.0052
   46 T(I,1,1)=EL(I)/(1.-ULT(I)*UTL(I))
      T(I,1,2)=EL(I)*UTL(I)/(1.-ULT(I)*UTL(I))
      T(I,1,3)=0.
      T(I,2,1)=T(I,1,2)
      T(I,2,2)=ET(I)/(1.-ULT(I)*UTL(I))
      T(I,2,3)=0.
      T(I,3,1)=0.
      T(I,3,2)=0.
```

```
      T(I,3,3)=GLT(I)
C CALCULATE ELEMENTS OF B MATRIX
      EM(I)=COSD(TH(I))
      EN(I)=SIND(TH(I))
   50 B(I,1,1)=T(I,1,1)*EM(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(EM(I)**2)*
     1(EN(I)**2)+T(I,2,2)*EN(I)**4
      B(I,1,2)=(T(I,1,1)+T(I,2,2)-4.*T(I,3,3))*(EM(I)**2)*(EN(I)**2)+T(I
     1,1,2)*(EM(I)**4+EN(I)**4)
      B(I,1,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EN(I)*EM(I)**3+(T(I,1,2)-
     1T(I,2,2)+2.*T(I,3,3))*EM(I)*EN(I)**3
      B(I,2,2)=T(I,1,1)*EN(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(EN(I)**2)*
     1(EM(I)**2)+T(I,2,2)*EM(I)**4
      B(I,2,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EM(I)*EN(I)**3+(T(I,1,2)-
     1T(I,2,2)+2.*T(I,3,3))*EN(I)*EM(I)**3
      B(I,3,3)=(T(I,1,1)+T(I,2,2)-2.*T(I,1,2)-2.*T(I,3,3))*(EN(I)**2)*
     1(EM(I)**2)+T(I,3,3)*(EM(I)**4+EN(I)**4)
      B(I,2,1)=B(I,1,2)
      B(I,3,1)=B(I,1,3)
      B(I,3,2)=B(I,2,3)
C CALCULATE ELEMENTS OF INVERSE OF D MATRIX
   70 DI(I,1,1)=EM(I)**2
      DI(I,1,2)=EN(I)**2
      DI(I,1,3)=-2.* EM(I)*EN(I)
      DI(I,2,1)=DI(I,1,2)
      DI(I,2,2)=DI(I,1,1)
      DI(I,2,3)=-DI(I,1,3)
      DI(I,3,1)=DI(I,2,3)/2.
      DI(I,3,2)=DI(I,1,3)/2.
   80 DI(I,3,3)=EM(I)**2-EN(I)**2
C CALCULATE ELEMENTS OF ALPHA MATRIX
      AX(I)=DI(I,1,1)*AL(I)+DI(I,1,2)*AT(I)
      AY(I)=DI(I,2,1)*AL(I)+DI(I,2,2)*AT(I)
   90 AXY(I)=DI(I,3,1)*AL(I)+DI(I,3,2)*AT(I)
  100 CONTINUE
      WRITE(6,710)(I,DI(I,1,1),DI(I,1,2),DI(I,1,3),I,DI(I,2,1),DI(I,2,2)
     1,DI(I,2,3),I,DI(I,3,1),DI(I,3,2),DI(I,3,3),I=1,N)
  710 FORMAT(1H1,30X,23HELEMENTS OF DI MATRIX  ///(10X,I2,5X,F17.8,5X,E1
     17.8,5X,F17.8))
      WRITE(6,720)(I,B(I,1,1),B(I,1,2),B(I,1,3),I,B(I,2,1),B(I,2,2),B(I,
     12,3),I,B(I,3,1),B(I,3,2),B(I,3,3),I=1,N)
  720 FORMAT(1H1,30X,22HELEMENTS OF B MATRIX  ///(10X,I2,5X,E17.8,5X,E17
     1.8,5X,E17.8))
      WRITE(6,740)(I,AX(I),AY(I),AXY(I),I=1,N)
  740 FORMAT(1H1,30X,26HELEMENTS OF ALPHA MATRIX  ///(10X,I2,5X,E17.8,5X
     1,E17.8,5X,E17.8))
C CALCULATE PROPERTIES OF N-PLY LAMINATE
C CALCULATE ELEMENTS OF B-BAR MATRIX
      BB(1,1)=0.
      BB(1,2)=0.
      BB(1,3)=0.
      BB(2,1)=0.
      BB(2,2)=0.
      BB(2,3)=0.
      BB(3,1)=0.
      BB(3,2)=0.
      BB(3,3)=0.
  120 DO 200 I=1,N
      DB1=B(I,1,1)*H(I)
      BB(1,1)=BB(1,1)+DB1
      DB2=B(I,1,2)*H(I)
```

```
      BB(1,2)=BB(1,2)+DB2
      DB3=B(I,1,3)*H(I)
      BB(1,3)=BB(1,3)+DB3
  140 DB4=B(I,2,1)*H(I)
      BB(2,1)=BB(2,1)+DB4
      DB5=B(I,2,2)*H(I)
      BB(2,2)=BB(2,2)+DB5
      DB6=B(I,2,3)*H(I)
      BB(2,3)=BB(2,3)+DB6
      DB7=B(I,3,1)*H(I)
  160 BB(3,1)=BB(3,1)+DB7
      DB8=B(I,3,2)*H(I)
      BB(3,2)=BB(3,2)+DB8
      DB9=B(I,3,3)*H(I)
      BB(3,3)=BB(3,3)+DB9
  200 CONTINUE
      WRITE(6,760) BB(1,1),BB(1,2),BB(1,3),BB(2,1),BB(2,2),BB(2,3),BB(3,
     11),BB(3,2),BB(3,3)
  760 FORMAT(1H1,30X,26HELEMENTS OF b BAR MATRIX ///(17X,F17.8,5X,F17.8
     1,5X,F17.8))
C CALCULATE ELEMENTS OF C MATRIX
      CX=0.
      DO 300 I=1,N
      DCX=(AX(I)*B(I,1,1)+AY(I)*B(I,1,2)+AXY(I)*B(I,1,3))*H(I)
      CX=CX+DCX
  300 CONTINUE
      CY=0.
      DO 400 I=1,N
      DCY=(AX(I)*B(I,2,1)+AY(I)*B(I,2,2)+AXY(I)*B(I,2,3))*H(I)
      CY=CY+DCY
  400 CONTINUE
      CXY=0.
      DO 500 I=1,N
      DCXY=(AX(I)*B(I,3,1)+AY(I)*B(I,3,2)+AXY(I)*B(I,3,3))*H(I)
      CXY=CXY+DCXY
  500 CONTINUE
C CALCULATE ELEMENTS OF INVERSE OF B-BAR MATRIX
      DBB=BB(1,1)*(BB(2,2)*BB(3,3)-BB(2,3)*BB(3,2))+BB(1,3)*(BB(2,1)*BB(
     13,2)-BB(2,2)*BB(3,1))+BB(1,2)*(BB(3,1)*BB(2,3)-BB(3,3)*BB(2,1))
      BI(1,1)=(BB(2,2)*BB(3,3)-BB(2,3)*BB(3,2))/DBB
  520 BI(1,2)=-(BB(1,2)*BB(3,3)-BB(1,3)*BB(3,2))/DBB
      BI(1,3)=(BB(1,2)*BB(2,3)-BB(1,3)*BB(2,2))/DBB
      BI(2,1)=-(BB(2,1)*BB(3,3)-BB(2,3)*BB(3,1))/DBB
      BI(2,2)=(BB(1,1)*BB(3,3)-BB(1,3)*BB(3,1))/DBB
  540 BI(2,3)=-(BB(1,1)*BB(2,3)-BB(1,3)*BB(2,1))/DBB
      BI(3,1)=(BB(2,1)*BB(3,2)-BB(2,2)*BB(3,1))/DBB
      BI(3,2)=-(BB(1,1)*BB(3,2)-BB(1,2)*BB(3,1))/DBB
  560 BI(3,3)=(BB(1,1)*BB(2,2)-BB(1,2)*BB(2,1))/DBB
C CALCULATE ELEMENTS OF ALPHA-BAR MATRIX
      AXB=BI(1,1)*CX+BI(1,2)*CY+BI(1,3)*CXY
      AYB=BI(2,1)*CX+BI(2,2)*CY+BI(2,3)*CXY
  580 AXYB=BI(3,1)*CX+BI(3,2)*CY+BI(3,3)*CXY
      WRITE(6,780)AXB,AYB,AXYB
  780 FORMAT(1H1,30X,28HELEMENTS OF ALPHA BAR MATRIX//22X,11HALPHA BAR X
     1,11X,11HALPHA BAR Y,11X,12HALPHA BAR XY//(17X,E17.8,5X,E17.8,5X,E1
     27.8))
C CALCULATE STRAINS DEVELOPED IN LAMINATE DUE TO UNIFORM TEMPERATURE
C     INCREASE DT
      EPX=DT*AXB
      EPY=DT*AYB
      EPXY=DT*AXYB
```

```
C CALCULATE STRESSES IN EACH PLY
  700 DO 600 I=1,N
      SX(I)=B(I,1,1)*(EPX-DT*AX(I))+B(I,1,2)*(EPY-DT*AY(I))+B(I,1,3)*
     1(EPXY-DT*AXY(I))
      SY(I)=B(I,2,1)*(EPX-DT*AX(I))+B(I,2,2)*(EPY-DT*AY(I))+B(I,2,3)*
     1(EPXY-DT*AXY(I))
      SXY(I)=B(I,3,1)*(EPX-DT*AX(I))+B(I,3,2)*(EPY-DT*AY(I))+B(I,3,3)*
     1(EPXY-DT*AXY(I))
      A(I,1,1)=T(I,1,1)*D(I,1,1)+T(I,1,2)*D(I,2,1)+T(I,1,3)*D(I,3,1)
      A(I,1,2)=T(I,1,1)*D(I,1,2)+T(I,1,2)*D(I,2,2)+T(I,1,3)*D(I,3,2)
      A(I,1,3)=T(I,1,1)*D(I,1,3)+T(I,1,2)*D(I,2,3)+T(I,1,3)*D(I,3,3)
      A(I,2,1)=T(I,2,1)*D(I,1,1)+T(I,2,2)*D(I,2,1)+T(I,2,3)*D(I,3,1)
      A(I,2,2)=T(I,2,1)*D(I,1,2)+T(I,2,2)*D(I,2,2)+T(I,2,3)*D(I,3,2)
      A(I,2,3)=T(I,2,1)*D(I,1,3)+T(I,2,2)*D(I,2,3)+T(I,2,3)*D(I,3,3)
      A(I,3,1)=T(I,3,1)*D(I,1,1)+T(I,3,2)*D(I,2,1)+T(I,3,3)*D(I,3,1)
      A(I,3,2)=T(I,3,1)*D(I,1,2)+T(I,3,2)*D(I,2,2)+T(I,3,3)*D(I,3,2)
      A(I,3,3)=T(I,3,1)*D(I,1,3)+T(I,3,2)*D(I,2,3)+T(I,3,3)*D(I,3,3)
      SL(I)=A(I,1,1)*EPX+A(I,1,2)*EPY+A(I,1,3)*EPXY-(T(I,1,1)*AL(I)+T(I,
     11,2)*AT(I))*DT
      ST(I)=A(I,2,1)*EPX+A(I,2,2)*EPY+A(I,2,3)*EPXY-(T(I,2,1)*AL(I)+T(I,
     12,2)*AT(I))*DT
      SLT(I)=A(I,3,1)*EPX+A(I,3,2)*EPY+A(I,3,3)*EPXY-(T(I,3,1)*AL(I)+T(I
     1,3,2)*AT(I))*DT
  600 CONTINUE
      WRITE(6,800)(I,SX(I),SY(I),SXY(I),I=1,N)
  800 FORMAT(1H1,30X,16HTHERMAL STRESSES//8X, 7HPLY NO., 8X,7HALPHA X,15
     1X,7HALPHA Y,15X,8HALPHA XY//(10X,I2,5X,E17.8,5X,E17.8,5X,E17.8))
      WRITE(6,900)(I,SL(I),ST(I),SLT(I),I=1,N)
  900 FORMAT(1H1,30X,29HTHERMAL STRESSES NATURAL AXES//8X,7HPLY NO., 8X,
     17HALPHA L,15X,7HALPHA T,15X,8HALPHA LT//(10X,I2,5X,E17.8,5X,E17.8,
     25X,E17.8))
      GO TO 5
      END
```


NUMBER
IDENTIFICATION
DESCRIPTION DO NOT KEY PUNCH
N
M
D T
A
73
80
N = Number of plies (20 max.)
M = Mat'l prop temp indicator { 1 - R.T. 2 - 220° F 3 - 350° F
DT = Temperature Increment
T H (1)
T H (2)
T H (3)
B
TH(1) = Angle of ply 1
TH(2) = Angle of ply 2
TH(3) = Angle of ply 3
(Maximum of 20)
Start next case

EXAMPLE PROBLEM

To illustrate the use of AC-40, consider a $[0/\pm45/0]_S$ laminate at room temperature with a $\Delta T = 10°$. For this case, on card A, there are four plies per set, so N = 4. M is set to 1 for room temperatures properties, and DT is set to 10.0 since $\Delta T = 10°$. For card B, only the angles 0.0, +45.0, -45.0, and 0.0 are input. The input data sheets and resulting output for this case are shown at the end of this section.

The results from the output printout are:

$\bar{\alpha}_x = 2.363 \times 10^{-6}$ in./in./°F

$\bar{\alpha}_y = 7.752 \times 10^{-6}$ in./in./°F

$\bar{\alpha}_{xy} = 0.0$

Ply	σ_x/10° F	σ_y/10° F	τ_{xy}/10° F	σ_L/10° F	σ_T/10° F	τ_{LT}/10 ° F
1	-3.7	-79.1	0	-3.7	-79.1	0
2	3.7	79.1	133.6	789.8	-137.1	-18.9
3	3.7	79.1	-133.6	789.8	-137.1	18.9
4	-3.7	-79.1	0	-3.7	-79.1	0

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	4	A	N
13	1		M
25	1 0 .		DT
37			
49		73 80	
61		1 0	
1	0 .	B	TH(1)
13	4 5 .		TH(2)
25	- 4 5 .		TH(3)
37	0 .		TH(4)
49		73 80	
61		2 0	
1			
13			
25			
37			
49		73 80	
61			
1			
13			
25			
37			
49		73 80	
61			

ELEMENTS OF ALPHA MATRIX

1	0.23199991E-05	0.10670000E-04	0.0
2	0.64950009E-05	0.64950009E-05	-0.41750000E-05
3	0.64950009E-05	0.64950009E-05	0.41750000E-05
4	0.23199991E-05	0.10670000E-04	0.0

ELEMENTS OF B BAR MATRIX

0.40758513E 06	0.86759750E 05	0.0
0.86759750E 05	0.12367444E 06	0.0
0.0	0.0	0.89434938E 05

ELEMENTS OF DI MATRIX

1	0.10000000E 01	0.0	0.0
1	0.0	0.10000000E 01	0.0
1	0.0	0.0	0.10000000E 01
2	0.50000018E 00	0.50000018E 00	-0.99999994E 00
2	0.50000018E 00	0.50000018E 00	0.99999994E 00
2	0.49999994E 00	-0.49999994E 00	0.0
3	0.50000018E 00	0.50000018E 00	0.99999994E 00
3	0.50000018E 00	0.50000018E 00	-0.99999994E 00
3	-0.49999994E 00	0.49999994E 00	0.0
4	0.10000000E 01	0.0	0.0
4	0.0	0.10000000E 01	0.0
4	0.0	0.0	0.10000000E 01

ELEMENTS OF B MATRIX

1	0.30019984E 08	0.57138356E 06	0.0
1	0.57138356E 06	0.27208750E 07	0.0
1	0.0	0.0	0.70000000E 06
2	0.91709110E 07	0.77709080E 07	0.68247770E 07
2	0.77709080E 07	0.91709110E 07	0.68247770E 07
2	0.68247770E 07	0.68247770E 07	0.78995250E 07
3	0.91709110E 07	0.77709080E 07	-0.68247770E 07
3	0.77709080E 07	0.91709110E 07	-0.68247770E 07
3	-0.68247770E 07	-0.68247770E 07	0.78995250E 07
4	0.30019984E 08	0.57138356E 06	0.0
4	0.57138356E 06	0.27208750E 07	0.0
4	0.0	0.0	0.70000000E 06

ELEMENTS OF ALPHA BAR MATRIX

ALPHA BAR X	ALPHA BAR Y	ALPHA BAR XY
0.23632065E-05	0.77521199E-05	0.0

THERMAL STRESSES

PLY NO.	ALPHA X	ALPHA Y	ALPHA XY
1	-0.37013893E 01	-0.79144974E 02	0.0
2	0.37009277E 01	0.79145752E 02	0.13361493E 03
3	0.37009277E 01	0.79145752E 02	-0.13361493E 03
4	-0.37013893E 01	-0.79144974E 02	0.0

THERMAL STRESSES NATURAL AXES

PLY NO.	ALPHA L	ALPHA T	ALPHA LT
1	-0.37014160E 01	-0.79144806E 02	0.0
2	0.78977881E 03	-0.13706204E 03	-0.18861176E 02
3	0.78977881E 03	-0.13706204E 03	0.18861176E 02
4	-0.37014160E 01	-0.79144806E 02	0.0

```
/ID 00,290,10=1,01,DAVIDSON,3883                                        12/02/69
/FTC NAME=AC2,LIST,DECK
RPS FORTRAN D COMPILER                                                  12/02/69
C
C     PROGRAM AC2 ... EXTENSIONAL A D FLEXURAL CONSTANTS FOR BALANCED,
C     SYMMETRICAL LAMINATES OF THE  RM (0/+-THETA/90)NS
C
C
C     THE INPUT DATA IS AS FOLLOWS.............
C          THE FIRST DATA CARD MUST CONTAIN THE LAMINATE ELASTIC CONSTANTS IN
C          THE FOLLOWING FORMAT...(5E12.0)
C          THE LAMINATE ELASTIC CONSTANTS ARE A,EL,ET,GLT,ULT WHERE.....
C            A IS A CONTROL CARD THAT INDICATES WHICH TYPE OF ELASTIC
C              CONSTANTS ARE TO BE USED. A=-1.0 IS FOR COMPRESSION
C              LOADINGS, A=0.0 IS FOR TENSION-COMPRESSION LOADINGS, AND
C              A=1.0 IS FOR TENSION LOADINGS.
C            EL IS THE LAMINA LONGITUDINAL YOUNGS MODULUS
C            ET IS THE LAMINA TRANSVERSE YOUNGS MODULUS
C            GLT IS THE LAMINA SHEAR MODULUS
C            ULT IS THE LAMINA POISSONS RATIO
C
C          THE SECOND AND FOLLOWING DATA CARDS MUST CONTAIN THE SET
C          DESCRIPTIONS IN THE FOLLOWING FORMAT...(I4,F8.0,3I4,F12.0,2I4)
C          THE SET DESCRIPTIONS INCLUDE O,H,IP1,IPT,IP2,THETA,IPS1, AND IPS2
C            WHERE.....
C            IO    IS THE TOTAL NUMBER OF SETS IN THE LAMINATE ( IT MUST ALWAYS
C               BE AN EVEN INTEGER)
C            H     IS THE NOMINAL PLY THICKNESS (FLOAT. PT.)
C            IP1   IS THE NUMBER OF INNER PLYS IN EACH IO SET IN THE ZERO OR
C               DEGREE DIRECTION DEPENDING ON IPS1 (INTEGER)
C            IPT IS THE NUMBER OF MIDDLE PLIES IN EACH O SET IN THE PLUS AND
C               MINUS THETA DEGREE DIRECTION (INTEGER)
C            IP2   IS THE NUMBER OF OUTER PLIES IN EACH IO SET IN THE ZERO OR
C               DEGREE DIRECTION DEPENDING ON IPS2 (INTEGER)
C            THETA IS THE PLUS AND MINUS ANGLE OF THE P2 PLIES (FLOAT. PT.)
C            IPS1 IS AN INDICATOR TO DETERMINE IF THE ANGLES ASSOCIATED WITH
C               P1 ARE ZERO OR NINETY DEGREES. IF IPS1 IS ZERO THEN THESE ANGLES
C               ARE ZERO DEGREES. IF IPS1 IS 90 THEN THESE ANGLES ARE 90 DEGREES
C            IPS2 IS THE SAME AS IPS1 EXCEPT THAT IPS2 REFERS TO P2
C
C
C
S.0001          DIMENSION D(6),QPR(4,3),DC(3)
S.0002          READ(5, 999) A,EL,ET,GLT,ULT
```

```
S.0003      999 FORMAT(5E12.0)
S.0004     2000 READ(5,1000)  IG,H,IP1,IPT,IP2,THETA,IPS1,IPS2
S.0005     1000 FORMAT(I4,F8.0,3I4,F12.0,2I4)
           C
S.0006           IF(A)7,5,3
S.0007         3 WRITE(6,4)
S.0008         4 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TENSION LOADINGS')
S.0009           GO TO 14
S.0010         5 WRITE(6,6)
S.0011         6 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS')
S.0012           GO TO 14
S.0013         7 WRITE(6,8)
S.0014         8 FORMAT('1...LAMINATE ELASTIC CONSTANTS FOR COMPRESSION LOADINGS')
S.0015        14 WRITE(6,9) EL,ET,GLT,ULT
S.0016         9 FORMAT('0 EL='F12.5,5X'ET='F12.5,5X'GLT='F12.5,5X'MULT='F8.4///)
           C
           C
S.0017           N=10
S.0018           P1=IP1
S.0019           PT=IPT
S.0020           P2=IP2
S.0021           RK=P1+2*PT+P2
S.0022           XN=N*RK
S.0023           R1=XN/2-RK
S.0024           R2=P1+P1
S.0025           R3=XN/2-P2
S.0026           T=XN*H
S.0027           IPT2=2*PT
           C
S.0028           I=0
S.0029           P=P1
S.0030           R=R1
S.0031       100 I=I+1
S.0032           V=N*PN/R*(2*(PN+1)*(2*PN+6*R-3*XN-2)+3*(2*R-XN-1)**2+RK*(N+2)*(3*
                1PN+4*R-3*XN+RK*(I+N))+1)
S.0033           IF(I-2) 110,120,130
S.0034       110 DC(I)=V
S.0035           P=2*PT
S.0036           R=R2
S.0037           GO TO 100
S.0038       120 DC(I)=V
S.0039           P=P2
S.0040           R=R3
S.0041           GO TO 100
```

```
S.0042      130 DC(I)=V
            C
S.0043          C=COSD(THETA)
S.0044          S=SIND(THETA)
S.0045          C2S2=(C*S)**2
S.0046          S2=S**2
S.0047          C2=C**2
S.0048          C2TS2=C2*S2
S.0049          C4=C**4
S.0050          S4=S**4
S.0051          C4PS4=C4+S4
            C
S.0052          UTL=(ET/EL)*ULT
S.0053          PSI=1-ULT*UTL
S.0054          Q11=EL/PSI
S.0055          Q12=Q11*UTL
S.0056          Q22=ET/PSI
S.0057          Q66=GLT
            C
S.0058          QPR(1,2)=Q11*C4+2*(Q12+2*Q66)*C2S2+Q22*S4
S.0059          QPR(2,1)=Q12
S.0060          QPR(2,2)=(Q11+Q22-4*Q66)*C2S2+Q12*(C4+S4)
S.0061          QPR(2,3)=Q12
S.0062          QPR(3,2)=Q11*S4+2*(Q12+2*Q66)*C2S2+Q22*C4
S.0063          QPR(4,1)=Q66
S.0064          QPR(4,2)=(Q11+Q22-2*Q12-2*Q66)*C2S2+Q66*(C4+S4)
S.0065          QPR(4,3)=Q66
S.0066          IF(IPS1-90)  140,145,145
S.0067      140 QPR(1,1)=Q11
S.0068          QPR(3,1)=Q22
S.0069          GO TO 150
S.0070      145 QPR(1,1)=Q22
S.0071          QPR(3,1)=Q11
S.0072      150 IF(IPS2-90)  155,160,160
S.0073      155 QPR(1,3)=Q11
S.0074          QPR(3,3)=Q22
S.0075          GO TO 165
S.0076      160 QPR(1,3)=Q22
S.0077          QPR(3,3)=Q11
S.0078      165 CONTINUE
            C
S.0079          AN=.6666666*(T/XN)**3.0
S.0080          DO 170 I=1,4
S.0081          D(I)=0.0
```

```
S.0082          DO 170 J=1,3
S.0083      170 D(I)=D(I)+QPR(I,J)*DC(J)*AM
                C
S.0084          Q16T=(Q11-Q12-2*Q66)*C**3*S+(Q12-Q22+2*Q66)*C*S**3
S.0085          Q26T=(Q11-Q12-2*Q66)*C*S**3+(Q12-Q22+2*Q66)*C**3*S
                C
S.0086          CON=2*(3*PI*(T/XN)**3*(-PI+XN/2-R2-R*R/4*(Q+2))
S.0087          D(5)=Q16T*CON
S.0088          D(6)=Q26T*CON
                C
S.0089          RATIO1=D(5)/D(1)
S.0090          RATIO2=D(5)/D(2)
S.0091          RATIO3=D(6)/D(1)
S.0092          RATIO4=D(6)/D(2)
                C
S.0093          A1=PI*QPR(1,1) +2*PI*QPR(1,2) +P2*QPR(1,3)
S.0094          A12=PI*QPR(2,1) +2*PI*QPR(2,2) +P2*QPR(2,3)
S.0095          A2=PI*QPR(3,1) +2*PI*QPR(3,2) +P2*QPR(3,3)
S.0096          A6=PI*QPR(4,1) +2*PI*QPR(4,2) +QPR(4,3)*P2
                C
S.0097          UYX=A12/A1
S.0098          UXY=A12/A2
S.0099          GXY=A6/RK
S.0100          EX=(A1*A2-A12**2)/A2/RK
S.0101          EY=(A1*A2-A12**2)/A1/RK
S.0102          AEF=(EX/EY)**.5
S.0103          PSII=(EX*EY)**.5/(1.0-UXY*UYX)
S.0104          B=1-UXY*UYX
S.0105          A11=EX*(1.0+EY/EX*2.0*UYX)/B
S.0106          A12=GXY-EX*UYX/B
                C
S.0107          WRITE(6,575)
S.0108      575 FORMAT('0...SET DESCRIPTION')
S.0109          WRITE(6,576)IO,H,T
S.0110      576 FORMAT('0   THERE ARE'I4  ' SETS OF PLIES WITH A NOMINAL PLY THICK
               1NESS OF'F7.4' INCHES.'/'    THE TOTAL LAMINATE THICKNESS IS'F10.4'
               2 INCHES.'/)
                C
S.0111          IF(IPS1-90)  577,579,579
                C
S.0112      577 WRITE(6,578)IP1
S.0113      578 FORMAT('   NUMBER OF INNER PLIES AT  0 DEGREES ='I3)
                C
S.0114          GO TO 585
```

```
S.0115      579 WRITE(6,580)  IP1
S.0116      580 FORMAT('   NUMBER OF INNER PLIES AT 90 DEGREES ='I3)
            C
S.0117      585 WRITE(6,590) THETA,IP12
S.0118      590 FORMAT('   NUMBER OF MIDDLE PLIES AT (+ AND -)'F7.2' DEGREES ='I3)
            C
S.0119           IF(IPS2-90)  600,620,620
            C
S.0120      600 WRITE(6,610)IP2
S.0121      610 FORMAT('   NUMBER OF OUTER PLIES AT  0 DEGREES ='I3//)
S.0122           GO TO 635
            C
S.0123      620 WRITE(6,630)IP2
S.0124      630 FORMAT('   NUMBER OF OUTER PLIES AT 90 DEGREES ='I3//)
            C
S.0125      635 WRITE(6,640)
S.0126      640 FORMAT('0...LAMINATE FLEXURAL CONSTANTS'/)
            C
S.0127           WRITE(6,650) D(1),D(2),D(3),A1,A12,D(4),D(5),D(6),A2,A6
S.0128      650 FORMAT('D11 ='E12.5,3X'D12 ='E12.5,3X'D22 ='E12.5,7X'A11 ='F12.5,
               13X'A12 ='E12.5/'D66 ='E12.5,3X'D16 ='E12.5,3X'D26 ='F12.5,7X'A22 =
               2'E12.5,3X'A66 ='E12.5//)
            C
S.0129           WRITE(6,660)
S.0130      660 FORMAT('0...RATIOS BETWEEN FLEXURAL CONSTANTS'/)
            C
S.0131           WRITE(6,670) RATIO1,RATIO2,RATIO3,RATIO4
S.0132      670 FORMAT('D16/D11 ='E12.5,5X'D16/D22 ='E12.5,5X'D26/D11 ='E12.5,5X'D
               126/D22 ='E12.5//)
            C
S.0133           WRITE(6,680)
S.0134      680 FORMAT('0...LAMINATE EXTENSIONAL CONSTANTS')
            C
S.0135           WRITE(6,13) EX,EY,GXY,UXY,UYX,ALP,PSI,A11,A12
S.0136       13 FORMAT('0'4X'EX='E12.5,5X'EY='E12.5,5X'GXY='E12.5,5X'MUXY='F7.4,5X
               1'MUYX='F7.4/' '1X'ALPHA=SQRT(EX/EY)='E12.5,13X'PSI=SQRT(EX*EY)/(1-
               2MUXY*MUYX)='E12.5/' '3X'A11=EX*(1+EY/EX+2*MUYX)='E12.5,5X'A12=GXY-
               3EX*MUYX/(1-MUXY*MUYX)='E12.5//)
            C
S.0137           BC=T**3/12/B
S.0138           D11A=BC*EX
S.0139           D12A=UYX*EX*BC
S.0140           D22A=BC*EY
S.0141           D66A=GXY*B*BC
```

```
                C
S.0142                WRITE(6,700)
S.0143          700 FORMAT('0...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL
                   1CONSTANTS'/)
                C
S.0144                WRITE(6,710)  D11A,D12A,D22A,D66A
S.0145          710 FORMAT('D11F =',F12.5,5X,'D12F =',E12.5,5X,'D22F =',F12.5,5X,'D66F =',F12
                   1.5)
                C
S.0146                GO TO 2000
S.0147                END
                     SIZE OF COMMON  00000     PROGRAM  06368
END OF COMPILATION  AC2
```

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0416 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 1

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12928E 03 D12 = 0.20975E 02 D22 = 0.32043E 02 A11 = 0.78382E 08 A12 = 0.16685E 08
D66 = 0.21746E 02 D16 =-0.76769E 01 D26 =-0.76769E 01 A22 = 0.23784E 08 A66 = 0.17199E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.59380E-01 D16/D22 =-0.36601E 00 D26/D11 =-0.59380E-01 D26/D22 =-0.36601E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.16669E 08 EY= 0.50580E 07 GXY= 0.42998E 07 MUXY= 0.7015 MUYX= 0.2129
ALPHA=SQRT(EX/EY)= 0.18154E 01 PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.10794E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXY-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.11756E 03 D12E = 0.25024E 02 D22E = 0.35671E 02 D66E = 0.25795E 02

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 4 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.1248 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 2
NUMBER OF MIDDLE PLIES AT (+ AND -) 45.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 90 DEGREES = 2

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.15476E 04 D12 = 0.45968E 03 D22 = 0.30215E 04 A11 = 0.83824E 08 A12 = 0.17827E 08
D66 = 0.48052E 03 D16 =-0.46062E 02 D26 =-0.46062E 02 A22 = 0.83824E 08 A66 = 0.18599E 08

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.29764E-01 D16/D22 =-0.10020E 00 D26/D11 =-0.29764E-01 D26/D22 =-0.10020E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.13339E 08 EY= 0.13339E 08 GXY= 0.30998E 07 MUXY= 0.2127 MUYX= 0.2127
ALPHA=SQRT(EX/EY)= 0.10000E 01 PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.13971E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXY-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.22630E 04 D12E = 0.48128E 03 D22E = 0.22630E 04 D66E = 0.50211E 03

...LAMINATE ELASTIC CONSTANTS FOR TEN-COMP LOADINGS

EL= 0.29900E 08 ET= 0.27100E 07 GLT= 0.70000E 06 MULT= 0.2100

...SET DESCRIPTION

THERE ARE 2 SETS OF PLIES WITH A NOMINAL PLY THICKNESS OF 0.0052 INCHES.
THE TOTAL LAMINATE THICKNESS IS 0.0312 INCHES.

NUMBER OF INNER PLIES AT 0 DEGREES = 1
NUMBER OF MIDDLE PLIES AT (+ AND -) 60.00 DEGREES = 2
NUMBER OF OUTER PLIES AT 0 DEGREES = 0

...LAMINATE FLEXURAL CONSTANTS

D11 = 0.12919E 02 D12 = 0.14606E 02 D22 = 0.43626E 02 A11 = 0.38312E 08 A12 = 0.12519E 06
D66 = 0.14932E 02 D16 =-0.31417E 01 D26 =-0.10155E 02 A22 = 0.38312E 08 A66 = 0.12899E 06

...RATIOS BETWEEN FLEXURAL CONSTANTS

D16/D11 =-0.24319E 00 D16/D22 =-0.21509E 00 D26/D11 =-0.78608E 00 D26/D22 =-0.69527E 00

...LAMINATE EXTENSIONAL CONSTANTS

EX= 0.11408E 08 EY= 0.11408E 08 GXY= 0.42998E 07 MUXY= 0.3266 MUYX= 0.3266
ALPHA=SQRT(EX/EY)= 0.10000E 01 PSI=SQRT(EX*EY)/(1-MUXY*MUYX)= 0.12771E 08
A11=EX*(1+EY/EX+2*MUYX)= 0.33884E 08 A12=GXY-EX*MUYX/(1-MUXY*MUYX)= 0.12862E 06

...LAMINATE FLEXURAL CONSTANTS FROM LAMINATE EXTENSIONAL CONSTANTS

D11E = 0.32322E 02 D12E = 0.10557E 02 D22E = 0.32322E 02 D66E = 0.10882E 02

```
/ID 00,290,1041,01,DAVIDSON,EX13864                                    12/05/69
/JOB TIME=10
/FIC NAME=AC5,LIST
BPS FORTRAN D COMPILER                                                 12/05/69
      C
      C     PROGRAM AC5 ... B/C PANEL STABILITY UNDER IN-PLANE BIAXIAL LOADING
      C
      C     THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
      C          AI1 = INVARIANT 1 = EX/(1-UMXY*UMYX)*(1+EY/EX+2*UMYX)
      C          AI2 = INVARIANT 2 = GXY - UMYX*EX/(1-UMXY*UMYX)
      C          ALF = SQRT(EX/EY)
      C          BETA = B = ALF*UMYX + 2*GAMMA
      C          D11 = EY*TF**3 /(2.*(1-UMXY*UMYX))
      C          D22 = D11*EY/EX
      C          D66 = TF**3*GXY/2
      C          DOX = C*GCX
      C          DOY = C*GCY
      C          GAMMA = G = GXY*(1-UMXY*UMYX)/SQRT(EX*EY)
      C          IA = TOTAL NUMBER OF A/B
      C          IC = TOTAL NUMBER OF HALF WAVELENGTHS IN Y-DIRECTION
      C          ID = TOTAL NUMBER OF HALF WAVELENGTHS IN X-DIRECTION
      C          IE = TOTAL NUMBER OF PHI, WHERE PHI = EY/NX
      C          IF = TOTAL NUMBER OF V WHERE V = PI**2*SQRT(D11*D22)/(B**2*DOX)
      C          PSI = SQRT(EX*EY)/(1-UMXY*UMYX)
      C          R = DOX/DOY
      C
S.0001          DIMENSION AOB(20), PHI(20), V(20), AK(20), APSI(20), TIT(18),
               1AAK(20),AAKAY(20), AKAY(20)
S.0002        1 READ(5,100) (TIT(I),I=1,18)
S.0003      100 FORMAT(18A4)
S.0004          READ(5,110) TNAOB, ALF, TNM, TNN, TNPHI, TNV, AI1, AI2, R, TNPSI
S.0005      110 FORMAT(6F12.0)
S.0006          A=ALF
S.0007          IA=TNAOB
S.0008          IC=TNM
S.0009          ID=TNN
S.0010          READ(5,110) (AOB(I),I=1,IA)
S.0011          IE=TNPHI
S.0012          IF(IE) 108,108,105
S.0013      105 READ(5,110) (PHI(I),I=1,IE)
S.0014      108 IP=TNV
S.0015          READ(5,110) (V(I),I=1,IP)
S.0016          L2=TNPSI
S.0017          READ(5,110) (APSI(I),I=1,L2)
```

```
S.0018          DO 300 L3=1,L2
S.0019          PSI=APSI(L3)
S.0020          GAMMA= AI1/(2.*PSI) + AI2/PSI -.5*A  -.5/A
S.0021          BETA= 1.5*AI1/PSI + 2.*AI2/PSI -1.5*A -1.5/A
S.0022          B=BETA
S.0023          G=GAMMA
S.0024          WRITE(6,120)
S.0025      120 FORMAT('1    BUCKLING COEFFICIENTS FOR  SIMPLY SUPPORTED ORTHOTROP
               1IC SANDWICH PLATES -  FLAT')
S.0026          WRITE(6,100) (TIT(I),I=1,18)
S.0027          WRITE(6,130) R, G, B, A, PSI
S.0028      130 FORMAT('0'5X'G''CX/G''CY='F5.2,5X'GAMMA='F8.4,5X'BETA='F8.3,5X'ALP
               1HA='F6.3,5X'PSI='F11.0)
S.0029          DO 300 I=1,IP
S.0030          WRITE(6,135) V(I)
S.0031      135 FORMAT(14HO DATA FOR  V=F8.4)
S.0032          IF(IE) 138,140,138
S.0033      138 WRITE(6,139)  (PHI(LL),LL=1,IE)
S.0034      139 FORMAT(' '26X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'PHI='F5.1,6X'
               1PHI='F5.1)
S.0035      140 CONTINUE
S.0036          WRITE(6,142)
S.0037      142 FORMAT('0'2X'A/B'5X'KX'6X'KY'5X'KX(1)/KY(1)'4X'KX(2)/KY(2)'4X'KX(3
               1)/KY(3)'4X'KX(4)/KY(4)'4X'KX(5)/KY(5)')
S.0038          DO 300 J=1,IA
S.0039          AB1=AOB(J)
S.0040          AB2=AB1**2
S.0041          AB3=AB1**3
S.0042          AB4=AB1**4
S.0043          AAKX= 1.0E+16
S.0044          AAKY=AAKX
S.0045          IF(IE) 145,150,145
S.0046      145 DO 148 JJ=1,IE
S.0047          AAKAY(JJ)=AAKX
S.0048      148 AAK(JJ)=AAKX
S.0049      150 CONTINUE
S.0050          DO 250 M=1,ID
S.0051          OM=M
S.0052          OM2=OM**2
S.0053          OM3=OM**3
S.0054          OM4=OM2**2
S.0055          DO 250 N=1,IC
S.0056          ON=N
S.0057          ON2=ON**2
```

```
S.0058            DM3=DM*DM2
S.0059            DM4=DM2**2
S.0060            A11= (A*DM3/AB3 +DM*DM2*B/AB1)*(-1.)
S.0061            A12= 1. + DM2*A*V(I)/AB2 +DM2*G*V(I)
S.0062            A13=(-1.)*(DM*DN*V(I)*R*(B-G)/AB1)
S.0063            A21=(DN3/A +DD*DM**2*B/AB2)
S.0064            A22= A13/R
S.0065            A23= 1.+R    *(DN2/A +G*DM2/AB2)* V(I)
S.0066            A31=(-1.)*(A*DM4/AB4 +2.*B*DM2*DM2/AB2 + DN4/A)
S.0067            A32= -DD /AB1* (DM2*A/AB2 +DM2*B)*V(I)
S.0068            A33= DM*R*V(I)*(DM2/A + DM2*B/AB2)
S.0069            AP31=(A33*(A11*A22-A12*A21) +A32*(A13*A21-A11*A23))/ (A12*A23 -A22
                 1*A13)
S.0070            AP= AP31-A31
S.0071            AKX=AP/(DM2/AB2)
S.0072            AKY=AP/DM2
S.0073            IF(AKX-AAKX) 160,170,170
S.0074        160 AAKX=AKX
S.0075        170 CONTINUE
S.0076            IF(AKY-AAKY) 175,180,180
S.0077        175 AAKY=AKY
S.0078        180 CONTINUE
S.0079            IF(IF) 185,280,185
S.0080        185 DO 195 L=1,IF
S.0081            AK(L)=AP/( DM2/AB2 +DN2*PHI(L))
S.0082            IF(AK(L)-AAK(L)) 190,195,195
S.0083        190 AAK(L)=AK(L)
S.0084        195 CONTINUE
S.0085            DO 198 L=1,IF
S.0086            AKAY(L)=AK(L)*PHI(L)
S.0087            IF(AKAY(L)-AAKAY(L)) 197,198,198
S.0088        197 AAKAY(L)=AKAY(L)
S.0089        198 CONTINUE
S.0090        250 CONTINUE
S.0091        280 PRINT(6,290) AOB(J), AAKX, AAKY, (AAK(L),AAKAY(L),L=1,IF)
S.0092        290 FORMAT(' 'F6.2,2F8.2, 5(F8.2'/'F6.2 ))
S.0093        300 CONTINUE
S.0094            GO TO 1
S.0095            END
                     SIZE OF COMMON  00000    PROGRAM  04116
END OF COMPILATION  AC5
```

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT (0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10.744000.

DATA FOR V= 0.0423

A/B	KX	KY	PHI= 0.1 KX(1)/KY(1)	PHI= 0.5 KX(2)/KY(2)	PHI= 1.0 KX(3)/KY(3)	PHI= 2.0 KX(4)/KY(4)	PHI= 10.0 KY(5)/KY(5)
1.00	4.17	3.91	3.79/ 0.38	2.78/ 1.39	2.09/ 2.09	1.39/ 2.78	0.38/ 3.79
2.00	4.17	1.17	3.35/ 0.34	1.56/ 0.78	0.94/ 0.94	0.52/ 1.04	0.11/ 1.14
3.00	4.06	0.79	3.31/ 0.33	1.29/ 0.65	0.71/ 0.71	0.37/ 0.75	0.08/ 0.78

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT
(0/0/+-45/90/90)2S LAMINATE AT ROOM TEMPERATURE

G'CX/G'CY= 2.00 GAMMA= 0.2219 BETA= 0.656 ALPHA= 1.000 PSI= 13971000.

DATA FOR V= 0.1896

			PHI= 0.1	PHI= 0.5	PHI= 1.0	PHI= 2.0	PHI= 10.0
A/B	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	2.27	2.19	2.06/ 0.21	1.51/ 0.76	1.13/ 1.13	0.76/ 1.51	0.21/ 2.06
2.00	2.27	1.00	2.06/ 0.21	1.33/ 0.67	0.80/ 0.80	0.44/ 0.89	0.10/ 0.97
3.00	2.27	0.84	2.06/ 0.21	1.34/ 0.67	0.75/ 0.75	0.40/ 0.79	0.08/ 0.83

BUCKLING COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES - FLAT (0/+-60)S LAMINATE AT ROOM TEMPERATURE

G'CX/G'CY= 2.00 GAMMA= 0.3367 BETA= 1.000 ALPHA= 1.000 PSI= 12771000.

DATA FOR V= 0.0368

			PHI= 0.1	PHI= 0.5	PHI= 1.0	PHI= 2.0	PHI= 10.0
A/B	KX	KY	KX(1)/KY(1)	KX(2)/KY(2)	KX(3)/KY(3)	KX(4)/KY(4)	KX(5)/KY(5)
1.00	3.60	3.60	3.28/ 0.33	2.40/ 1.20	1.80/ 1.80	1.20/ 2.40	0.33/ 3.28
2.00	3.60	1.44	3.28/ 0.33	1.92/ 0.96	1.15/ 1.15	0.64/ 1.28	0.14/ 1.41
3.00	3.60	1.15	3.28/ 0.33	1.87/ 0.94	1.03/ 1.03	0.54/ 1.09	0.11/ 1.15

```
/ID 00,290,1041,01,DAVIDSON,3883                                        12/05/69
/FTC NAME=AC7,LIST,DECK
BPS FORTRAN D COMPILER                                                  12/05/69
                C
                C     PROGRAM AC7 ... ANALYSIS OF SIMPLY-SUPPORTED ORTHOTROPIC SANDWICH
                C     FLAT PANELS UNDER PRESSURE OR PRESSURE PLUS IN-PLANE LOADS
                C
                C     THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.........
                C          A11 = INVARIANT 1 = EX*(1+EY/EX+2*MUYX)/(1-MUXY*MUYX)
                C          A12 = INVARIANT 2 = GXY - MUYX*EX/(1-MUXY*MUYX)
                C          AKX = BUCKLING COEFFICIENT IN THE X-DIRECTION FOUND FROM PROGRAM AC5
                C          AKY = BUCKLING COEFFICIENT IN THE Y-DIRECTION FOUND FROM PROGRAM AC5
                C          ALPHA = ALF = SQRT(EX/EY)
                C          BETA = B = ALPHA*MUYX + 2*GAMMA
                C          D11 = EX*TF*C**2/(2*(1-MUXY*MUYX))
                C          D22 = D11*(EY/EX)
                C          D66 = TF*C**2*GXY/2
                C          DOX = C*GCX
                C          DOY = C*GCY
                C          GAMMA = G = GXY*(1-MUXY*MUYX)/SQRT(EX*EY)
                C          IA = TOTAL NUMBER OF A/B(PANEL ASPECT RATIOS)
                C          IF = TOTAL NUMBER OF PHI, WHERE PHI = NY/NX
                C          IP = TOTAL NUMBER OF V, WHERE V = PI**2*SQRT(D11*D22)/(R**2*DOX)
                C          MI = TRUNCATION INTEGER FOR SERIES SUMMATION
                C          PSI = SQRT(EX*EY)/(1-MUXY*MUYX)
                C          R = DOX/DOY = GCX/GCY
                C          RX = NX/NXCR
                C          RY = NY/NYCR
                C          XI = SQRT(D11*D22)
                C
 S.0001                 DIMENSION AOB(20),XOA(20),YOB(20),PHI(20),V(20),QX(20),QY(20),
                       1 ANX(20),ANY(20),ANXY(20),P(20),AOX(20),AOY(20),SY(20),CY(20)
                       2,APSI(20),TIT(18),AAKX(20),AAKY(20)
 S.0002                1 READ(5,7) (TIT(I),I=1,18)
 S.0003                7 FORMAT(18A4)
 S.0004                  READ(5,2)TNAOB,ALF,TNM,TNV,A11,A12,RX,RY,R,TNPSI,TNXOA,TNYOB
 S.0005                2 FORMAT(6F12.0)
 S.0006                  A=ALF
 S.0007                  D1=0.0166424
 S.0008                  IA=TNAOB
 S.0009                  MI=TNM
 S.0010                  MA=TNXOA
 S.0011                  MR=TNYOB
 S.0012                3 READ(5,2)  (AOB(I),AAKX(I),AAKY(I),I=1,IA)
```

```
S.0013          6 IP=TNV
S.0014            READ(5,2)(V(I,,I=1,IP)
S.0015            L2=TNPSI
S.0016            READ(5,2)(APSI(I),I=1,L2)
S.0017            READ(5,2)(XOA(I),I=1,MA)
S.0018            READ(5,2)(YOB(I),I=1,MB)
S.0019            DO 100 L3=1,L2
S.0020            DO 100 J=1,IA
S.0021            AB1=AOB(J)
S.0022            AKX=AAKX(J)
S.0023            AKY=AAKY(J)
S.0024            AB2=AB1**2
S.0025            AB3=AB1**3
S.0026            AB4=AB1**4
S.0027            PSI=APSI(L3)
S.0028            GAMMA=(AI1/(2.*PSI) + AI2/PSI -.5*A  -.5/A)
S.0029            BETA=(1.5*AI1/PSI + 2.*AI2/PSI -1.5*A -1.5/A)
S.0030            B=BETA
S.0031            G=GAMMA
S.0032            WRITE(6,8)
S.0033          8 FORMAT('1')
S.0034            WRITE(6,7) (TIT(I),I=1,18)
S.0035         10 WRITE(6,11)
S.0036         11 FORMAT('0   DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SU
                  1PPORTED ORTHOTROPIC  SANDWICH PLATES')
S.0037         15 WRITE(6,16)R,G,B,A,PSI
S.0038         16 FORMAT('0'5X'G''CX/G''CY='F5.2,5X'GAMMA='F8.4,5X'BETA='F8.3,5X'ALP
                  1HA='F6.3,5X'PSI='F11.0)
S.0039         17 WRITE(6,18) AOB(J),RX,RY,AKX,AKY
S.0040         18 FORMAT('0'5X'A/B ='F5.2,5X'NX/NXCR ='F6.3,5X'NY/NYCR ='F6.3,5X'KX
                  1='F5.2,5X'KY ='F5.2)
S.0041         25 DO 100 I=1,IP
S.0042         26 WRITE(6,27)V(I)
S.0043         27 FORMAT('0'2X'...DATA FOR V ='F8.4)
S.0044            DO 100 J1=1,MA
S.0045            C1=3.1416*XOA(J1)
S.0046        250 WRITE(6,251)XOA(J1)
S.0047        251 FORMAT(25H   COEFFICIENTS FOR X/A=F6.3 )
S.0048        505 WRITE(6,506)
S.0049        506 FORMAT('0              W*XI/Q*B**4      MX/Q*B**2      MY/Q*B**2     M
                  1XY/Q*B**2     QX/Q*B      QY/Q*B')
S.0050            DO 199 J2=1,MB
S.0051            W(J2)=.0
S.0052            AMX(J2)=.0
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```
S.0053          AMY(J2)=.0
S.0054          AMXY(J2)=.0
S.0055          AOX(J2)=.0
S.0056          AOY(J2)=.0
S.0057      199 CONTINUE
S.0058       30 DO 90 M=1,MI,2
S.0059          DO 90 N=1,M,2
S.0060          OM=M
S.0061          ON=N
S.0062      352 SY=SIN (OM*C1)
S.0063          CX=COS (OM*C1)
S.0064          DO 200 K=1,MB
S.0065          SY(K)=SIN (YOB(K)*3.1416*ON)
S.0066          CY(K)=COS (YOB(K)*3.1416*ON)
S.0067      200 CONTINUE
S.0068          OM2=OM**2
S.0069          OM3=OM**3
S.0070          OM4=OM2**2
S.0071          ON2=ON**2
S.0072          ON3=ON*ON2
S.0073          ON4=ON2**2
S.0074          A11= (A*OM3/AB3 +ON*ON2*B/AB1)*(-1.)
S.0075          A12= 1. + OM2*A*V(I)/AB2 +OM2*G*V(I)
S.0076          A13=(-1.)*(OM*ON*V(I)*R*(R-G)/AB1)
S.0077          A21=(OM3/A +ON*OM**2*B/AB2)
S.0078          A22= A13/R
S.0079          A23= 1.+R    *(ON2/A +G*OM2/AB2)* V(I)
S.0080          A31=(-1.)*(A*OM4/AB4 +2.*R*OM2*OM2/AB2 + ON4/A)
S.0081          A32= -OM /AB1* (OM2*A/AB2 +ON2*B)*V(I)
S.0082          A33= ON*R*V(I)*(OM2/A + ON2*B/AB2)
S.0083          AMN=(A12*A23-A22*A13)/(A33*(A11*A22-A12*A21)+A32*(A13*A21-A11*A23)
               1+(-A31-AKX*RX*OM2/AB2-AKY*RY*OM2)*(A12*A23-A22*A13))*D1/(OM*ON)
S.0084          DO=A22*A13-A12*A23
S.0085          ALF2=(A11*A22-A12*A21)/DO
S.0086          ALF1=(A11*A23-A13*A21)/DO
S.0087          S=V(I)
S.0088          BMX=- 9.8696        *AMN*((OM*S*ALF1/AB1-OM2/AB2)*A+(B-2.*G)*(R*S*
               1ON*ALF2-ON2))
S.0089          BMY=- 9.8696        *AMN*(1./A*(ON*ALF2*R*S -ON2)+(B-2.*G)*(ALF1*OM
               1*S/AB1-OM**2/AB2) )
S.0090          BMXY=G* 9.8696     *AMN*(ON*ALF1*S +ALF2*R*OM*S -2.*OM*ON/AB1)
S.0091          BOX=- 31.0065    *AMN*OM*ON*(1./(ON*AB1)*((B-2.*G)*(ON*ALF2*R*S
               1-OM2)+A*(ALF1*OM*S/AB1 -OM2/AB2)) +G*ON*(ALF1*S/OM +ALF2*R*S/(ON*
               2AB1) -2./AB1))
```

```
S.0092           BOY=- 31.0065   *AMN*   (G*OM*(ALF1*S/OM+ALF2*R*S/(ON*AB1)-2./AB1)
                1*(1./AB1)+(1./OM)*((ON*ALF2*R*S-ON2)/A+(B-2.*G)*(ALF1*OM*S/AB1
                2-OM2/AB2)))*OM*ON
S.0093           DO 300 J2=1,MB
S.0094           AMX(J2)=AMX(J2)+BMX*SX*SY(J2)
S.0095           AMY(J2)=AMY(J2)+BMY*SX*SY(J2)
S.0096           AMXY(J2)=AMXY(J2)+BMXY*CX*CY(J2)
S.0097           AQX(J2)=AQX(J2)+BQX*CX*SY(J2)
S.0098           AQY(J2)=AQY(J2)+BQY*SX*CY(J2)
S.0099           W(J2)=W(J2)+AMN*   SX*SY(J2)
S.0100       300 CONTINUE
S.0101           OM1=M
S.0102           IF(OM1-OM)90 ,400,90
                 C
                 C.....LOOP TO PICK UP ANM TERM
                 C
S.0103       400 IF(M-N)401,403,401
S.0104       401 OM=N
S.0105           ON=M
S.0106           GO TO 352
S.0107       403 CONTINUE
S.0108       500 WRITE(6,501) M,N
S.0109       501 FORMAT(4HO M=,I2,3X,2HN=,I2)
S.0110     . 510 DO 520 J2=1,MB
S.0111       511 WRITE(6,512)YOB(J2),W(J2),AMX(J2),AMY(J2),AMXY(J2),AQX(J2),AQY(J2)
S.0112       512 FORMAT(5HOY/B=F6.3,F9.6,F13.3,F14.3,F14.3,F13.3,F11.3)
S.0113       520 CONTINUE
S.0114        90 CONTINUE
S.0115       100 CONTINUE
S.0116           GO TO 1
S.0117           END
                    SIZE OF COMMON  00000    PROGRAM  06380
END OF COMPILATION  AC7
```

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PS1= 10794000.

A/B = 1.00 MX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.17 KY = 3.91

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.028	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.328	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.337	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.367	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.031	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.368	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.032	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.378	0.0

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.188
Y/B= 0.500	0.003988	0.077	0.032	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.218
Y/B= 0.500	0.003816	0.068	0.027	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.238
Y/B= 0.500	0.003839	0.070	0.028	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.244
Y/B= 0.500	0.003832	0.069	0.028	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.251
Y/B= 0.500	0.003835	0.069	0.028	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G*CX/G*CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 2.00 MX/NXCR = 0.0 NY/NYCR = 0.0 KX = 4.17 KY = 1.17

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.050	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.336	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.055	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.395	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.056	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.436	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.056	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.448	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.056	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.451	0.0

COEFFICIENTS FOR X/A= 0.500

	W=X1/(D*B**4	MX/(D*B**2	MY/(D*B**2	MXY/(D*B**2	QX/(D*B	QY/(D*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.348
Y/B= 0.500	0.014188	0.110	0.085	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.353
Y/B= 0.500	0.013603	0.091	0.075	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.364
Y/B= 0.500	0.013671	0.095	0.077	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.365
Y/B= 0.500	0.013653	0.094	0.076	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.366
Y/B= 0.500	0.013660	0.094	0.077	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE , UNDER PRESSURE ONLY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 3.00 MX/MXCR = 0.0 NY/NYCR = 0.0 KX = 4.06 KY = 0.79

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.049	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.285	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.058	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.370	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.059	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.421	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.059	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.442	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.059	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.459	0.0

COEFFICIENTS FOR X/A= 0.500

	MXXI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.421
Y/B= 0.500	0.021051	0.116	0.117	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.399
Y/B= 0.500	0.019594	0.087	0.101	-0.000	0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.442
Y/B= 0.500	0.019796	0.096	0.105	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.436
Y/B= 0.500	0.019745	0.092	0.103	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.449
Y/B= 0.500	0.019763	0.094	0.104	-0.000	0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 1.00 NX/NXCR = 0.500 NY/NYCR = 0.500 KX = 2.09 KY = 2.09

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.055	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.656	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.059	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.661	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.694	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.695	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.060	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.705	0.0

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.376
Y/B= 0.500	0.007989	0.154	0.065	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.419
Y/B= 0.500	0.007774	0.143	0.058	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.443
Y/B= 0.500	0.007800	0.145	0.060	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.450
Y/B= 0.500	0.007792	0.144	0.059	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.457
Y/B= 0.500	0.007795	0.145	0.059	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 2.00 MX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.94 KY = 0.94

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.101	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.674	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.107	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.742	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.107	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.785	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.107	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.798	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.107	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.811	0.0

COEFFICIENTS FOR X/A= 0.500

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.697
Y/B= 0.500	0.028423	0.220	0.170	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.703
Y/B= 0.500	0.027754	0.198	0.159	-0.000	-0.000	0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.737
Y/B= 0.500	0.027826	0.203	0.161	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.738
Y/B= 0.500	0.027807	0.201	0.160	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.748
Y/B= 0.500	0.027814	0.202	0.160	-0.000	-0.000	-0.000

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE, UNDER PRESSURE PLUS NX AND NY

DEFLECTION,MOMENT AND SHEAR COEFFICIENTS FOR SIMPLY SUPPORTED ORTHOTROPIC SANDWICH PLATES

G'CX/G'CY= 2.00 GAMMA= 0.3985 BETA= 1.184 ALPHA= 1.815 PSI= 10794000.

A/B = 3.00 MX/NXCR = 0.500 NY/NYCR = 0.500 KX = 0.71 KY = 0.71

...DATA FOR V = 0.0423

COEFFICIENTS FOR X/A= 0.0

	W*XI/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	-0.098	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.568	0.0
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	-0.108	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.674	0.0
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.729	0.0
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.751	0.0
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	-0.110	0.0	0.0
Y/B= 0.500	0.0	0.0	0.0	0.000	0.769	0.0

COEFFICIENTS FOR X/A= 0.500

	W*X1/Q*B**4	MX/Q*B**2	MY/Q*B**2	MXY/Q*B**2	QX/Q*B	QY/Q*B
M= 1 N= 1						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.840
Y/B= 0.500	0.042011	0.231	0.233	-0.000	-0.000	-0.000
M= 3 N= 3						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.810
Y/B= 0.500	0.040766	0.197	0.215	-0.000	-0.000	-0.000
M= 5 N= 5						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.856
Y/B= 0.500	0.040487	0.206	0.218	-0.000	-0.000	-0.000
M= 7 N= 7						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.850
Y/B= 0.500	0.040432	0.202	0.217	-0.000	-0.000	-0.000
M= 9 N= 9						
Y/B= 0.0	0.0	0.0	0.0	0.000	0.0	0.864
Y/B= 0.500	0.040451	0.204	0.218	-0.000	-0.000	-0.000

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/ID 00,290,1041,01,DAVIDSON,EXT3883                                    12/05/69
/JOB TIME=10
/FTC NAME=AC11,LIST,DECK
BPS FORTRAN D COMPILER                                                 12/05/69
      C
      C     PROGRAM AC11 ... H/C PANEL STABILITY FOR IN-PLANE SHEAR LOADING
      C
      C     THE INPUT DATA AND SELECTED CONSTANTS ARE AS FOLLOWS.....
      C          A = PANEL WIDTH ALONG X-DIRECTION
      C          AI1 = INVARIANT 1 = EX*(1+EY/EX+2*MUYX)/(1-MUXY*MUYX)
      C          AI2 = INVARIANT 2 = GXY - MUYX*EX/(1-MUXY*MUYX)
      C          ALPHA = SQRT(EX/EY)
      C          AOB = PANEL ASPECT RATIO A/B
      C          B = PANEL LENGTH ALONG Y-DIRECTION
      C          C = CORE DEPTH
      C          CNTL = CONTROL CARD FOR DESIGNATING TYPE OF INPUT TO BE USED
      C                 (=0 FOR MATL. PROPERTIES, OR =1 FOR MATL. PARAMETERS)
      C          EIGMIN = EIGENVALUE ITERATION SOLUTION THAT TAKES THE SQRT
      C                   OF THE ABS VALUE OF SUCCEEDING EIGENVALUES
      C          GCX = CORE SHEAR MODULUS IN THE X-DIRECTION
      C          GCY = CORE SHEAR MODULUS IN THE Y-DIRECTION
      C          PSI = SQRT(EX*EY)/(1-MUXY*MUYX)
      C          Q = NUMBER OF TERMS IN THE SERIES (MAXIMUM OF 13)
      C          R = GCX/GCY
      C          TF = THICKNESS OF ONE FACE SHEET
      C          TITLE = ANY DESIRED ONE-LINE TITLE IN COLS.1-72
      C          V = PI**2*PSI*TF*(C+TF)/(2*GCX*B**2)
      C          XNIT = NUMBER OF ITERATIONS DESIRED IN THE SOLUTION
      C
S.0001            DIMENSION  H( 85, 85), TITLE(18), BB(2)              00000100
S.0002         10 WRITE(6,50)                                          00000115
S.0003         50 FORMAT(1H1)                                          00000117
S.0004            READ(5,100) TITLE
S.0005        100 FORMAT(18A4)                                         00000140
S.0006            WRITE(6,100) TITLE                                   00000150
      C.....READ MATERIAL PROPERTIES                                   00000160
S.0007            READ(5,110) CNTL,Q
S.0008        110 FORMAT(6F12.0)                                       00000180
S.0009            IF(CNTL) 101,109,101
      C.....READ PARAMETRIC PROPETIES
S.0010        101 READ(5,110)  ALPHA,PSI,V,R,AI1,AI2,AOB,C,TF,XNIT
S.0011            CC1=AI1/PSI-ALPHA-1.0/ALPHA
S.0012            BETA=1.5*CC1+2.*AI2/PSI
S.0013            GAMA=0.5*CC1+AI2/PSI
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S.0014          UMXY=ALPHA*(BETA-2.0*GAMA)
S.0015          UMYX=UMXY/ALPHA**2
S.0016          AMDA=1.0-UMXY*UMYX
S.0017          EX=ALPHA*PSI*AMDA
S.0018          EY=EX/ALPHA**2
S.0019          GXY=A12+UMYX*EX/AMDA
S.0020          SX=V*C/(C+TF)/AOB**2
S.0021          WRITE(6,115) ALPHA,PSI,V,R
S.0022      115 FORMAT(1H04X,28HTHE MATERIAL PARAMETERS ARE,//10X,6HALPHA=F6.3,5X,
                   14HPSI=E12.5,5X,2HV=F7.4,5X,8HGCX/GCY=F6.3)
S.0023          WRITE(6,116) AOB,C,TF
S.0024      116 FORMAT(1H0,4X,28HTHE GEOMETERY PROPERTIES ARE,//12X,4HA/B=F7.3,6X,
                   12HC=F7.4,10X,3HTF=F7.4)
S.0025          GO TO 131
          C.....READ MATERIAL PROPETIES
S.0026      109 READ(5,110) EX,EY,GXY,UMXY,GCX,GCY
          C.....READ PANEL GEOMETRY                                          00000190
S.0027      112 READ(5,110) A,B,C,TF,XNIT                                    00000200
S.0028          AOB=A/B                                                      00000300
S.0029          UMYX=EY/EX*UMXY                                              00000230
S.0030          ALPHA=(EX/EY)**.5                                            00000330
S.0031          AMDA=1.0-UMYX*UMXY                                           00000335
S.0032          PSI=(EX*EY)**.5/AMDA
S.0033          GAMA=GXY/PSI                                                 00000350
S.0034          BETA=ALPHA*UMYX+2.0*GAMA                                     00000360
S.0035          P=C+TF
S.0036          T=PSI*TF
S.0037          D=T*P**2/2.0
S.0038          SX=C*D*9.8696/(P*A)**2/GCX                                   00000400
S.0039          R=GCX/GCY                                                    00000410
S.0040          V=SX*P*AOB**2/C
          C.....WRITE INPUT DATA                                             00000240
S.0041      118 WRITE(6,120) EX,GCX,EY,GCY,GXY,UMXY,UMYX                     00000250
S.0042      120 FORMAT(1H0,4X,28HTHE MATERIAL PROPERTIES ARE //10X,15HFACE SHEET E
                   1X =E13.5,10X, 9HCORE GX =E13.5/21X,4HEY =E13.5,15X,4HGY =E13.5/20X00000270
                   25HGXY =E13.5/19X,6HMUXY =F8.4/19X,6HMUYX =F8.4)          00000280
                C                                                            00000290
S.0043          WRITE(6,130) A,B,C,TF,AOB                                    00000305
S.0044      130 FORMAT(1H0,4X,22HTHE PANEL GEOMETRY IS //10X,16HPANEL LENGTH A =F800000310
                   1.3,5X,15HPANEL WIDTH B =F8.3/12X,14HCORE DEPTH C =F8.3, 5X,25HFACE00000315
                   2 SHEET THICKNESS TF =F8.3/21X,5HA/B =F8.3)               00000320
S.0045          WRITE(6,125) ALPHA,PSI,V,R
S.0046      125 FORMAT(1H04X,28HTHE MATERIAL PARAMETERS ARE,//10X,6HALPHA=F6.3,5X,
                   14HPSI=E12.5,5X,2HV=F7.4,5X,8HGCX/GCY  6.3)
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                C.....CALCULATE CONSTANTS                                       00000325
S.0047        131 SY=R*SX
S.0048            P=C+TF                                                        00000370
S.0049            T=PSI*TF                                                      00000380
S.0050            D=T*P**2/2.0                                                  00000390
S.0051            AOB2=AOB**2                                                   00000440
S.0052            AOB4=AOB2**2                                                  00000450
S.0053            IQ=Q+.001                                                     00000460
                C.....XLMN CONSTANTS                                            00000470
S.0054            XLMN1=GAMA*AOB2                                               00000480
S.0055            XLMN2=  R  /ALPHA*AOB2                                        00000490
S.0056            XLMN3=  R  *GAMA                                              00000500
                C.....XKMN CONSTANTS                                            00000510
S.0057            XKMN1=2.0*BETA*AOB2                                           00000520
S.0058            XKMN2=AOB4/ALPHA                                              00000530
                C.....FMN CONSTANT                                              00000540
S.0059            XFMN1=1.0-BETA**2                                             00000550
                C.....VMN CONSTANTS                                             00000560
S.0060            XVMN1=AOB2*SX                                                 00000570
S.0061            XVMN2=SX*SY                                                   00000580
                C.....DF CONSTANT                                               00000590
S.0062            DF=TF**3/12.0*PSI                                             00000600
S.0063            DFQD=DF/D                                                     00000610
                C.....CALCULATE VARIABLE CONSTANTS AFTER CHECKING (M+N)         00000620
                                                                                00000630
S.0064            IP=1
S.0065        135 I=0                                                           00000640
S.0066            NIT=XNIT                                                      00000645
S.0067        140 DO 200 M=1,IQ                                                 00000650
S.0068            A1=M                                                          00000660
S.0069            HA=3.24227*A1                                                 00000665
S.0070            A2=A1**2                                                      00000670
S.0071            A4=A2**2                                                      00000680
S.0072            IF(M-2*(M/2)) 145,150,145                                     00000690
S.0073        145 NS=IP                                                         00000700
S.0074            GO TO 155                                                     00000710
S.0075        150 NS=3-IP                                                       00000720
S.0076        155 DO 200 N=NS,IQ,2                                              00000730
S.0077            I=I+1                                                         00000740
S.0078            B1=N                                                          00000750
S.0079            HB=HA*B1                                                      00000760
S.0080            B2=B1**2                                                      00000770
S.0081            B4=B2**2                                                      00000780
                C.....CALCULATE VARIABLE CONSTANTS                              00000785
S.0082            XLMN    =ALPHA*A2+XLMN1*B2+XLMN2*B2+XLMN3*A2                  00000790
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S.0083            XKMN    =ALPHA*A4+XKMN1*A2*R2+XKMN2*B4                                  00000800
S.0084            FMN     =  XI*MN1+GAMA*XKMN                                             00000810
S.0085            VMN   =(XKMN+(XVMN1*B2+SY*A2)*FMN)/(1.0+XLMN*SX+XVMN2*FMN)            00000820
S.0086            VMNI    =DFOO*XKMN                                                      00000840
S.0087            AA=(2.0*VMNI+VMN)/AOB                                                   00000850
                C.....CHECK TO SEE IF (M+R) AND (N+S) ARE ODD OR EVEN                   00000855
                                                                                        00000860
S.0088            J=0                                                                   00000870
S.0089            DO 190 IR=1,IQ                                                        00000880
S.0090            RI=IR                                                                 00000885
S.0091            HR=HR*RI                                                              00000890
S.0092            RI2=RI**2                                                             00000895
S.0093            IF(IR-2*(IR/2)) 160,170,160                                           00000900
S.0094       160  ISS=IP                                                               00000910
S.0095            GO TO 180                                                             00000920
S.0096       170  ISS=3-IP                                                            00000925
S.0097       180  DO 190 IS=ISS,IQ,2                                                   00000930
S.0098            SI=IS                                                                 00000932
S.0099            HC=HR*SI                                                              00000935
S.0100            SI2=SI**2                                                             00000940
S.0101            J=J+1                                                                 00000942
S.0102            IF(M+IR-2*((M+IR)/2)) 182,187,182                                     00000943
S.0103       182  IF(N+IS-2*((N+IS)/2)) 185,187,185                                    00000945
S.0104       185  H(I,J)=-HC   /(A2-RI2)/(B2-SI2)/AA                                   00000947
S.0105            GO TO 190                                                             00000948
S.0106       187  H(I,J)=0.0                                                           00000950
S.0107       190  CONTINUE                                                            00000955
S.0108       200  CONTINUE                                                            00000960
S.0109            IO2=(IO**2+2-IP)/2                                                    00000965
S.0110            DELTA=.001                                                            00000966
S.0111            CALL EIGMIN(IO2,DELTA,H,BB(IP),NIT)                                   00000967
S.0112            WRITE(6,217) NIT                                                      00000970
S.0113       217  FORMAT(1HO,5X,4HNIT=I6/)                                            00000980
S.0114            IF(IP-2)220,240,240                                                   00000990
S.0115       220  IP=2                                                                01001100
S.0116            GO TO 135
                C.....PRINT OUT EVEN AND ODD KS  WHICH ARE BB(1) AND BB(2)
S.0117       240  BB(1)=BB(1)/AOR2
S.0118            BB(2)=BB(2)/AOR2
S.0119            IF(BB(1)-BB(2)) 245,250,250
S.0120       245  XKS=BB(1)
S.0121            GO TO 255
S.0122       250  XKS=BB(2)
S.0123       255  IF(CNTL) 256,260,256
S.0124       256  XKNXCR=XKS*9.8696*0
```

```
S.0125            WRITE(6,258) XKNXCR
S.0126       258 FORMAT('0'5X'NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) ='
                 1E13.5'/B**2')
S.0127            GO TO 280
S.0128       260 XNCRS=XKS*9.8686*D/B**2
S.0129            WRITE(6,265) IO,BB(1),BB(2)                                    00001110
S.0130       265 FORMAT(1HO,38H****THE VALUES FOR KS  WHEN THERE ARE I3,22H CMN COE00001120
                 1FFICIENTS ARE,/20X,10H KS EVEN =E13.5,10X, 9H KS ODD =E13.5)   00001130
S.0131            WRITE(6,270) XNCRS
S.0132       270 FORMAT('0'5X'NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) ='
                 1E13.5)
S.0133       280 GO TO 10
S.0134            END                                                            00001150
                      SIZE OF COMMON  00000    PROGRAM  33640
END OF COMPILATION  AC11
```

```
/FTC NAME=EIGMIN,LIST,DECK                                                    12/05/69
BPS FORTRAN D COMPILER                                                        12/05/69
 S.0001          SUBROUTINE EIGMIN(NN,DELTA,BINV,ALAMIN,NIT)                  00002100
 S.0002          DIMENSION C( 85),B1( 85),BINV( 85,85)
          C                                                                   00002120
 S.0003          II=1                                                         00002125
 S.0004        1 DO 2 I=1,NN                                                  00002130
 S.0005          C(I)=1.0                                                     00002140
 S.0006        2 CONTINUE                                                     00002150
 S.0007          M=1                                                          00002160
 S.0008        3 DO 5 I=1,NN                                                  00002170
 S.0009          B1(I)=0.0                                                    00002180
 S.0010        4 DO 5 J=1,NN                                                  00002190
 S.0011          B1(I)=B1(I)+BINV(I,J)*C(J)                                   00002200
 S.0012        5 CONTINUE                                                     00002210
 S.0013          C1=ABS(B1(1))                                                00002220
 S.0014          ALAMDA=C1                                                    00002260
 S.0015        8 DO 9 I=1,NN                                                  00002270
 S.0016          C(I)=B1(I)/C1                                                00002280
 S.0017        9 CONTINUE                                                     00002290
 S.0018          IF(M-1) 11,10,11                                             00002300
 S.0019       10 M=M+1                                                        00002310
 S.0020          BLAMDA=ALAMDA                                                00002320
 S.0021          C4=(BLAMDA*ALAMDA)**.5
 S.0022          GO TO 3                                                      00002330
 S.0023       11 C2=(ALAMDA*BLAMDA)**.5
 S.0024          C3=ABS(C2-C4)
 S.0025          IF(C3-DELTA)  14,14,12
 S.0026       12 BLAMDA=ALAMDA                                                00002360
 S.0027          C4=C2
 S.0028          II=II+1                                                      00002365
 S.0029          IF(II-NIT) 13,14,14                                          00002366
 S.0030       13 GO TO 3                                                      00002370
 S.0031       14 NIT=II                                                       00002375
 S.0032          ALAMIN=1.0/C2                                                00002390
 S.0033          RETURN                                                       00002400
 S.0034          END
                 SIZE OF COMMON  00000    PROGRAM  01574
END OF COMPILATION  EIGMIN
```

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PROPERTIES ARE

FACE SHEET EX =	0.16670E 08	CORE GX =	0.12600E 06
EY =	0.50580E 07	GY =	0.63000E 05
GXY =	0.43000E 07		
MUXY =	0.7015		
MUYX =	0.2128		

THE PANEL GEOMETRY IS

PANEL LENGTH A =	15.000	PANEL WIDTH B =	15.000
CORE DEPTH C =	0.500	FACE SHEET THICKNESS TF =	0.042
A/B =	1.000		

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

NIT= 7

NIT= 11

****THE VALUES FOR KS WHEN THERE ARE 13 CMN COEFFICIENTS ARE,

KS EVEN = 0.78484E 01 KS ODD = 0.84673E 01

NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) = 0.22670E 05

(0/+-45/0)S LAMINATE AT ROOM TEMPERATURE

THE MATERIAL PARAMETERS ARE,

ALPHA= 1.815 PSI= 0.10794E 08 V= 0.0423 GCX/GCY= 2.000

THE GEOMETERY PROPERTIES ARE

A/B= 1.000 C= 0.5000 TF= 0.0416

NIT= 7

NIT= 11

NSCR=KS * PI**2 * PSI * TF * (C+TF)**2/(2 * B**2) = 0.51021E 07/B**2

```
/ID 00,290,1041,01,DAVIDSON,EXT3883                                    12/10/69
/FTC NAME=AC40,LIST,DECK
BPS FORTRAN D COMPILER                                                 12/10/69
            C
            C     PROGRAM AC40...N-PLY LAMINATE THERMOELASTIC ANALYSIS PROGRAM
            C
            C        INPUT DATA IS AS FOLLOWS.....
            C              N = NUMBER OF PLIES
            C              M = MATL. PROP. TEMP. INDICATOR....M=1 FOR R.T., M=2 FOR 350F,
            C                  M=3 FOR 350F
            C              DT = TEMPERATURE INCREMENT IN DEGREES F
            C              TH(I) = INDIVIDUAL PLY ANGLES THETA(I)
            C
 S.0001             DIMENSION EL(20),FT(20),GLT(20),ULT(20),UTL(20),AL(20),AT(20),   00010010
                   1   H(20),A(20,3,3),SL(20),ST(20),SLT(20),TH(20)                   00010020
 S.0002             DIMENSION T(20,3,3),EM(20),EN(20),R(20,3,3),DI(20,3,3),AX(20),   00010075
                   1   AXY(20),BB(3,3),SX(20),SY(20),SXY(20),BI(3,3),AY(20)          00010077
 S.0003             DIMENSION D(20,3,3)                                              00010079
 S.0004             EQUIVALENCE (D(1),DI(1))                                         00010081
 S.0005           5 READ(5,10) N,M,DT                                                00010030
 S.0006          10 FORMAT(2I12, F12.0)                                              00000040
            C TH(I) = ANGLE BETWEEN NATURAL AXES OF I-TH PLY AND LAMINATE AXES      00010044
 S.0007          15 READ(5,20) (TH(I),I=1,N)                                        00010070
 S.0008          20 FORMAT(6F12.0)                                                  00010090
            C PROPERTIES OF INDIVIDUAL LAMINA                                       00010100
 S.0009          40 DO 100 I=1,N                                                    00010102
            C CALCULATE ELEMENTS OF T MATRIX                                        00010104
 S.0010             IF(M-2)42,43,44                                                00010106
            C ROOM TEMPERATURE PROPERTIES                                           00010110
 S.0011          42 EL(I)=29900000.                                                00010112
 S.0012             FT(I)=2710000.                                                 00010114
 S.0013             GLT(I)=700000.                                                 00010116
 S.0014             ULT(I)=.21                                                     00010118
 S.0015             UTL(I)=ULT(I)*FT(I)/EL(I)                                      00010120
 S.0016             AL(I)=.00000232                                                00010122
 S.0017             AT(I)=.00001067                                                00010124
 S.0018             H(I)=.0052                                                     00010125
 S.0019             GO TO 46
            C 220 DEGREE PROPERTIES
 S.0020          43 EL(I)=29900000.
 S.0021             ET(I)=1420000.
 S.0022             GLT(I)=377100.
 S.0023             ULT(I)=.21
 S.0024             UTL(I)=ULT(I)*FT(I)/EL(I)
```

```
S.0025          AL(I)=.000002414                                                00010126
S.0026          AT(I)=.00001394                                                 00010128
S.0027          H(I)=.0052                                                      00010130
S.. 028         GO TO 46                                                        00010132
      C 350 DEGREE PROPERTIES                                                   00010134
S.0029       44 EL(I)=29900000.                                                 00010136
S.0030          ET(I)=1130000.                                                  00010138
S.. 031         GLT(I)=300000.                                                  00010140
S.0032          ULT(I)=.21                                                      00010142
S.0033          UTL(I)=ULT(I)*ET(I)/EL(I)                                       00010144
S.0034          AL(I)=.00000283                                                 00010146
S.. 035         AT(I)=.0000197R                                                 00010148
S.0036          H(I)=.0052                                                      00010150
S.0037       46 T(I,1,1)=EL(I)/(1.-ULT(I)*UTL(I))                               00010160
S.0038          T(I,1,2)=EL(I)*UTL(I)/(1.-ULT(I)*UTL(I))                        00010170
S.0039          T(I,1,3)=0.                                                     00010180
S.0040          T(I,2,1)=T(I,1,2)                                               00010190
S.0041          T(I,2,2)=ET(I)/(1.-ULT(I)*UTL(I))                               00010200
S.0042          T(I,2,3)=0.                                                     00010220
S.0043          T(I,3,1)=0.                                                     00010230
S.0044          T(I,3,2)=0.                                                     00010240
S.. 045         T(I,3,3)=GLT(I)                                                 00010250
      C CALCULATE ELEMENTS OF B MATRIX                                          00010260
S.0046          EM(I)=COSD(TH(I))                                               00010270
S.0047          EN(I)=SIND(TH(I))                                               00010280
S.. 048      50 B(I,1,1)=T(I,1,1)*EM(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(EM(I)**2)*00010290
     1(EN(I)**2)+T(I,2,2)*EN(I)**4                                              00010300
S.0049          B(I,1,2)=(T(I,1,1)+T(I,2,2)-4.*T(I,3,3))*(EM(I)**2)*(EN(I)**2)+T(I00010310
     1,1,2)*(EM(I)**4+EN(I)**4)                                                 00010320
S.0050          B(I,1,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EN(I)*EM(I)**3+(T(I,1,2)-00010330
     1T(I,2,2)+2.*T(I,3,3))*EM(I)*EN(I)**3                                      00010340
S.0051          B(I,2,2)=T(I,1,1)*EN(I)**4+2.*(T(I,1,2)+2.*T(I,3,3))*(EN(I)**2)*00010350
     1(EM(I)**2)+T(I,2,2)*EM(I)**4                                              00010360
S.0052          B(I,2,3)=(T(I,1,1)-T(I,1,2)-2.*T(I,3,3))*EM(I)*EN(I)**3+(T(I,1,2)-00010370
     1T(I,2,2)+2.*T(I,3,3))*EN(I)*EM(I)**3                                      00010380
S.0053          B(I,3,3)=(T(I,1,1)+T(I,2,2)-2.*T(I,1,2)-2.*T(I,3,3))*(EN(I)**2)*00010390
     1(EM(I)**2)+T(I,3,3)*(EM(I)**4+EN(I)**4)                                   00010400
S.0054          B(I,2,1)=B(I,1,2)                                               00010420
S.0055          B(I,3,1)=B(I,1,3)                                               00010430
S.. 056         B(I,3,2)=B(I,2,3)                                               00010440
      C CALCULATE ELEMENTS OF INVERSE OF D MATRIX
S.0057       70 DI(I,1,1)=EM(I)**2
S.0058          DI(I,1,2)=EN(I)**2
S.0059          DI(I,1,3)=-2.* EM(I)*EN(I)
```

```
S.0060          DI(1,2,1)=DI(I,1,2)                                          00010450
S.0061          DI(I,2,2)=DI(I,1,1)                                          00010460
S.0062          DI(I,2,3)=-DI(I,1,3)                                         00010470
S.0063          DI(I,3,1)=DI(I,2,3)/2.                                       00010480
S.0064          DI(I,3,2)=DI(I,1,3)/2.                                       00010490
S.0065       80 DI(I,3,3)=EM(I)**2-EN(I)**2                                  00010500
     1      C CALCULATE ELEMENTS OF ALPHA MATRIX                            00010510
S.0066          AX(I)=DI(I,1,1)*AL(I)+DI(I,1,2)*AT(I)                        00010520
S.0067          AY(I)=DI(I,2,1)*AL(I)+DI(I,2,2)*AT(I)                        00010530
S.0068       90 AXY(I)=DI(I,3,1)*AL(I)+DI(I,3,2)*AT(I)                       00010532
S.0069      100 CONTINUE                                                     00010534
S.0070          WRITE(6,710)(I,DI(I,1,1),DI(I,1,2),DI(I,1,3),I,DI(I,2,1),DI(I,2,2)00010536
               1,DI(I,2,3),I,DI(I,3,1),DI(I,3,2),DI(I,3,3),I=1,N)           00010538
S.0071      710 FORMAT(1H1,30X,23HELEMENTS OF DI MATRIX  ///(10X,I2,5X,E17.8,5X,E100010540
               17.8,5X,E17.8))                                              00010542
S.0072          WRITE(6,720)(I,B(I,1,1),B(I,1,2),B(I,1,3),I,B(I,2,1),B(I,2,2),B(I,00010545
               12,3),I,B(I,3,1),B(I,3,2),B(I,3,3),I=1,N)                    00010546
S.0073      720 FORMAT(1H1,30X,22HELEMENTS OF B MATRIX  ///(10X,I2,5X,E17.8,5X,E1700010548
               1.8,5X,E17.8))                                               00010550
S.0074          WRITE(6,740)(I,AX(I),AY(I),AXY(I),I=1,N)                     00010552
S.0075      740 FORMAT(1H1,30X,26HELEMENTS OF ALPHA MATRIX  ///(10X,I2,5X,E17.8,5X00010554
               1,E17.8,5X,E17.8))                                           00010556
          C CALCULATE PROPERTIES OF N-PLY LAMINATE                          00010568
          C CALCULATE ELEMENTS OF B-BAR MATRIX                              00010570
S.0076          BB(1,1)=0.                                                   00010580
S.0077          BB(1,2)=0.                                                   00010590
S.0078          BB(1,3)=0.                                                   00010600
S.0079          BB(2,1)=0.                                                   00010610
S.0080          BB(2,2)=0.                                                   00010620
S.0081          BB(2,3)=0.                                                   00010630
S.0082          BB(3,1)=0.                                                   00010640
S.0083          BB(3,2)=0.                                                   00010650
S.0084          BB(3,3)=0.                                                   00010660
S.0085      120 DO 200 I=1,N                                                 00010670
S.0086          DB1=B(I,1,1)*H(I)                                            00010680
S.0087          BB(1,1)=BB(1,1)+DB1                                          00010690
S.0088          DB2=B(I,1,2)*H(I)                                            00010700
S.0089          BB(1,2)=BB(1,2)+DB2                                          00010710
S.0090          DB3=B(I,1,3)*H(I)                                            00010720
S.0091          BB(1,3)=BB(1,3)+DB3                                          00010730
S.0092      140 DB4=B(I,2,1)*H(I)                                            00010740
S.0093          BB(2,1)=BB(2,1)+DB4                                          00010750
S.0094          DB5=B(1,2,2)*H(I)                                            00010760
S.0095          BB(2,2)=BB(2,2)+DB5                                          00010770
```

```
S.0096          DB6=B(I,2,3)*H(I)                                                00010780
S.0097          BB(2,3)=BB(2,3)+DB6                                              00010790
S.0098          DB7=B(I,3,1)*H(I)                                                00010800
S.0099      160 BB(3,1)=BB(3,1)+DB7                                              00010810
S.0100          DB8=B(I,3,2)*H(I)                                                00010820
S.0101          BB(3,2)=BB(3,2)+DB8                                              00010830
S.0102          DB9=B(I,3,3)*H(I)                                                00010840
S.0103          BB(3,3)=BB(3,3)+DB9                                              00010850
S.0104      200 CONTINUE                                                         00010855
S.0105          WRITE(6,760) BB(1,1),BB(1,2),BB(1,3),BB(2,1),BB(2,2),BB(2,3),BB(3,00010857
               11),BB(3,2),BB(3,3)                                               00010859
S.0106      760 FORMAT(1H1,30X,26HELEMENTS OF B BAR MATRIX  ///(17X,E17.8,5X,E17.800010861
               1,5X,E17.8))                                                      00010863
            C CALCULATE ELEMENTS OF C MATRIX                                     00010870
S.0107          CX=0.                                                            00010880
S.0108          DO 300 I=1,N                                                     00010890
S.0109          DCX=(AX(I)*B(I,1,1)+AY(I)*B(I,1,2)+AXY(I)*B(I,1,3))*H(I)         00010900
S.0110          CX=CX+DCX                                                        00010910
S.0111      300 CONTINUE                                                         00010920
S.0112          CY=0.                                                            00010930
S.0113          DO 400 I=1,N                                                     00010940
S.0114          DCY=(AX(I)*B(I,2,1)+AY(I)*B(I,2,2)+AXY(I)*B(I,2,3))*H(I)         00010950
S.0115          CY=CY+DCY                                                        00010960
S.0116      400 CONTINUE                                                         00010970
S.0117          CXY=0.                                                           00010980
S.0118          DO 500 I=1,N                                                     00010990
S.0119          DCXY=(AX(I)*B(I,3,1)+AY(I)*B(I,3,2)+AXY(I)*B(I,3,3))*H(I)        00011000
S.0120          CXY=CXY+DCXY                                                     00011010
S.0121      500 CONTINUE                                                         00011020
            C CALCULATE ELEMENTS OF INVERSE OF B-BAR MATRIX                      00011030
S.0122          DBB=BB(1,1)*(BB(2,2)*BB(3,3)-BB(2,3)*BB(3,2))+BB(1,3)*(BB(2,1)*BB(00011040
               13,2)-BB(2,2)*BB(3,1))+BB(1,2)*(BB(3,1)*BB(2,3)-BB(3,3)*BB(2,1))  00011050
S.0123          BI(1,1)=(BB(2,2)*BB(3,3)-BB(2,3)*BB(3,2))/DBB                    00011060
S.0124      520 BI(1,2)=-(BB(1,2)*BB(3,3)-BB(1,3)*BB(3,2))/DBB                   00011070
S.0125          BI(1,3)=(BB(1,2)*BB(2,3)-BB(1,3)*BB(2,2))/DBB                    00011080
S.0126          BI(2,1)=-(BB(2,1)*BB(3,3)-BB(2,3)*BB(3,1))/DBB                   00011090
S.0127          BI(2,2)=(BB(1,1)*BB(3,3)-BB(1,3)*BB(3,1))/DBB                    00011100
S.0128      540 BI(2,3)=-(BB(1,1)*BB(2,3)-BB(1,3)*BB(2,1))/DBB                   00011110
S.0129          BI(3,1)=(BB(2,1)*BB(3,2)-BB(2,2)*BB(3,1))/DBB                    00011120
S.0130          BI(3,2)=-(BB(1,1)*BB(3,2)-BB(1,2)*BB(3,1))/DBB                   00011130
S.0131      560 BI(3,3)=(BB(1,1)*BB(2,2)-BB(1,2)*BB(2,1))/DBB                    00011140
            C CALCULATE ELEMENTS OF ALPHA-BAR MATRIX                             00011150
S.0132          AXB=BI(1,1)*CX+BI(1,2)*CY+BI(1,3)*CXY                            00011160
S.0133          AYB=BI(2,1)*CX+BI(2,2)*CY+BI(2,3)*CY                             00011170
```

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S.0134      580 AXYB=BI(3,1)*CX+BI(3,2)*CY+BI(3,3)*CXY                                    00011180
S.0135          WRITE(6,780)AXB,AYB,AXYB                                                  00011182
S.0136      780 FORMAT(1H1,30X,28HELEMENTS OF ALPHA BAR MATRIX//22X,11HALPHA BAR X00011184
               1,11X,11HALPHA BAR Y,11X,12HALPHA BAR XY//(17X,E17.8,5X,E17.8,5X,E100011185
               27.8))                                                                    00011186
            C CALCULATE STRAINS DEVELOPED IN LAMINATE DUE TO UNIFORM TEMPERATURE         00011195
            C     INCREASE DT                                                           00011200
S.0137          EPX=DT*AXB                                                            00011210
S.0138          EPY=DT*AYB                                                            00011220
S.0139          EPXY=DT*AXYB                                                          00011230
            C CALCULATE STRESSES IN EACH PLY                                          00011240
S.0140      700 DO 600 I=1,N                                                          00011250
S.0141          SX(I)=B(I,1,1)*(EPX-DT*AX(I))+B(I,1,2)*(EPY-DT*AY(I))+B(I,1,3)*       00011260
               1(EPXY-DT*AXY(I))                                                      00011270
S.0142          SY(I)=B(I,2,1)*(EPX-DT*AX(I))+B(I,2,2)*(EPY-DT*AY(I))+B(I,2,3)*       00011280
               1(EPXY-DT*AXY(I))                                                      00011290
S.0143          SXY(I)=B(I,3,1)*(EPX-DT*AX(I))+B(I,3,2)*(EPY-DT*AY(I))+B(I,3,3)*      00011300
               1(EPXY-DT*AXY(I))                                                      00011310
S.0144          A(I,1,1)=T(I,1,1)*D(I,1,1)+T(I,1,2)*D(I,2,1)+T(I,1,3)*D(I,3,1)        00011312
S.0145          A(I,1,2)=T(I,1,1)*D(I,1,2)+T(I,1,2)*D(I,2,2)+T(I,1,3)*D(I,3,2)        00011314
S.0146          A(I,1,3)=T(I,1,1)*D(I,1,3)+T(I,1,2)*D(I,2,3)+T(I,1,3)*D(I,3,3)        00011316
S.0147          A(I,2,1)=T(I,2,1)*D(I,1,1)+T(I,2,2)*D(I,2,1)+T(I,2,3)*D(I,3,1)        00011318
S.0148          A(I,2,2)=T(I,2,1)*D(I,1,2)+T(I,2,2)*D(I,2,2)+T(I,2,3)*D(I,3,2)        00011320
S.0149          A(I,2,3)=T(I,2,1)*D(I,1,3)+T(I,2,2)*D(I,2,3)+T(I,2,3)*D(I,3,3)        00011322
S.0150          A(I,3,1)=T(I,3,1)*D(I,1,1)+T(I,3,2)*D(I,2,1)+T(I,3,3)*D(I,3,1)        00011324
S.0151          A(I,3,2)=T(I,3,1)*D(I,1,2)+T(I,3,2)*D(I,2,2)+T(I,3,3)*D(I,3,2)        00011326
S.0152          A(I,3,3)=T(I,3,1)*D(I,1,3)+T(I,3,2)*D(I,2,3)+T(I,3,3)*D(I,3,3)        00011328
S.0153          SL(I)=A(I,1,1)*EPX+A(I,1,2)*EPY+A(I,1,3)*EPXY-(T(I,1,1)*AL(I)+T(I,00011330
               11,2)*AT(I))*DT                                                        00011332
S.0154          ST(I)=A(I,2,1)*EPX+A(I,2,2)*EPY+A(I,2,3)*EPXY-(T(I,2,1)*AL(I)+T(I,00011334
               12,2)*AT(I))*DT                                                        00011336
S.0155          SLT(I)=A(I,3,1)*EPX+A(I,3,2)*EPY+A(I,3,3)*EPXY-(T(I,3,1)*AL(I)+T(I00011338
               1,3,2)*AT(I))*DT                                                       00011340
S.0156      600 CONTINUE                                                              00011345
S.0157          WRITE(6,800)(I,SX(I),SY(I),SXY(I),I=1,N)                              00011350
S.0158      800 FORMAT(1H1,30X,16HTHERMAL STRESSES//8X,7HPLY NO.,8X,7HALPHA X,15X00011360
               1X,7HALPHA Y,15X,8HALPHA XY//(10X,I2,5X,E17.8,5X,E17.8,5X,E17.8)) 00011361
S.0159          WRITE(6,900)(I,SL(I),ST(I),SLT(I),I=1,N)                          00011380
S.0160      900 FORMAT(1H1,30X,29HTHERMAL STRESSES NATURAL AXES//8X,7HPLY NO.,8X,00011390
               17HALPHA L,15X,7HALPHA T,15X,8HALPHA LT//(10X,I2,5X,E17.8,5X,E17.8,00011391
               25X,E17.8))                                                            00011392
S.0161          GO TO 5                                                               00011450
S.0162          END                                                                   00011450
                   SIZE OF COMMON   00000     PROGRAM   10020
```

ELEMENTS OF ALPHA MATRIX .

1	0.23199991E-05	0.10670000E-04	0.0
2	0.64950009E-05	0.64950009E-05	-0.41750000E-05
3	0.64950009E-05	0.64950009E-05	0.41750000E-05
4	0.23199991E-05	0.10670000E-04	0.0

ELEMENTS OF B BAR MATRIX

0.40758513E 06	0.86759750E 05	0.0
0.86759750E 05	0.12367444E 06	0.0
0.0	0.0	0.89434938E 05

ELEMENTS OF DI MATRIX

1	0.10000000E 01	0.0	0.0
1	0.0	0.10000000E 01	0.0
1	0.0	0.0	0.10000000E 01
2	0.50000018E 00	0.50000018E 00	-0.99999994E 00
2	0.50000018E 00	0.50000018E 00	0.99999994E 00
2	0.49999994E 00	-0.49999994E 00	0.0
3	0.50000018E 00	0.50000018E 00	0.99999994E 00
3	0.50000018E 00	0.50000018E 00	-0.99999994E 00
3	-0.49999994E 00	0.49999994E 00	0.0
4	0.10000000E 01	0.0	0.0
4	0.0	0.10000000E 01	0.0
4	0.0	0.0	0.10000000E 01

ELEMENTS OF B MATRIX

1	0.30019984E 08	0.57138356E 06	0.0
1	0.57138356E 06	0.27208750E 07	0.0
1	0.0	0.0	0.70000000E 06
2	0.91709110E 07	0.77709080E 07	0.68247770E 07
2	0.77709080E 07	0.91709110E 07	0.68247770E 07
2	0.68247770E 07	0.68247770E 07	0.78995250E 07
3	0.91709110E 07	0.77709080E 07	-0.68247770E 07
3	0.77709080E 07	0.91709110E 07	-0.68247770E 07
3	-0.68247770E 07	-0.68247770E 07	0.78995250E 07
4	0.30019984E 08	0.57138356E 06	0.0
4	0.57138356E 06	0.27208750E 07	0.0
4	0.0	0.0	0.70000000E 06

ELEMENTS OF ALPHA BAR MATRIX

ALPHA BAR X	ALPHA BAR Y	ALPHA BAR XY
0.23632065E-05	0.77521199E-05	0.0

THERMAL STRESSES

PLY NO.	ALPHA X	ALPHA Y	ALPHA XY
1	-0.37013893E 01	-0.79144974F 02	0.0
2	0.37009277E 01	0.79145752F 02	0.13361493E 03
3	0.37009277E 01	0.79145752F 02	-0.13361493E 03
4	-0.37013893E 01	-0.79144974F 02	0.0

THERMAL STRESSES NATURAL AXES

PLY NO.	ALPHA L	ALPHA T	ALPHA LT
1	-0.37014160E 01	-0.79144806E 02	0.0
2	0.78977881E 03	-0.13706204E 03	-0.18861176E 02
3	0.78977881E 03	-0.13706204E 03	0.18861176E 02
4	-0.37014160E 01	-0.79144806E 02	0.0

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
A	Card A	A = Lamina data type (-1.0 = comp, 0.0 = ten-comp, 1.0 = ten.)
E L		EL = Longitudinal Young's modulus for one ply
E T		ET = Transverse Young's modulus for one ply
G L T		GLT = Shear modulus for one ply
U L T	73 80	ULT = Poisson's ratio for one ply
I Q H	Card B	IQ = Total number of sets (even integer)
I P 1 I P T I P 2		H = nominal ply thickness (Fl. Pt.)
T H E T A		IP1 = Number of inner plies in each IQ set (integer)
I P S 1 I P S 2		IPT = Number of plus theta plies in each IQ set (integer)
	73 80	IP2 = Number of outer plies in each IQ set (integer)
		THETA = ± Angle of the middle plies (must be negative if the
		+ theta degree ply is closest to the laminate surface...
		...Fl. Pt.)
		IPS1 = Angle indicator for the inner plies. Make 0 for 0° and
		90 for 90°. (integer can only be 0 or 90)
	73 80	IPS2 = Same as IPS1 except that it refers to the outer IP2 plies
	73 80	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.0	Card A	A = 0.0 for average tension-compression data)
13	29900000.		EL
25	2710000.		ET
37	700000.		GLT
49	.210	73 80	ULT
61		1 0	
1	2 .0052	Card B	Number of sets / Nominal ply thickness
13	1 1 1		IP1 / IPT / IP2
25	45.0		Theta
37	0 0		Angle of IP1 plies / Angle of IP2 plies
49		73 80	
61		2 0	$[0/\pm45/0]_S$
1	4 .0052	Card B	
13	2 1 2		
25	455.0		
37	0 90		
49		73 80	
61		3 0	$[0_2/\pm45/90_2]_{2S}$
1	2 .0052	Card B	
13	1 1 0		
25	60.0		
37	0 0		
49		73 80	
61		4 0	$[0/\pm60]_S$

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	T I T	CARD A (73–80)	TIT = Any desired one-card title in columns 1-72
13			
25			
37			
49			
61			
1	T N A O B	CARD B (73–80)	TNAOB = Total number of (a/b)'s to be considered
13	A L F		ALF = $\sqrt{E_x/E_y}$
25	T N N		TNN = Total number of n
37	T N M		TNM = Total number of m
49	T N P H I		TNPHI = Total number of ϕ
61	T N V		TNV = Total number of V
1	A I 1	CARD C (73–80)	AI1 = Invariant 1 = $E_x\,(1 + E_y/E_x + 2\nu_{yx})/(1 - \nu_{xy}\,\nu_{yx})$
13	A I 2		AI2 = Invariant 2 = $G_{xy} - E_x\,\nu_{yx}/(1 - \nu_{xy}\,\nu_{yx})$
25	R		.. = $D_{qx}/D_{qy} = G'_{cx}/G'_{cy}$
37	T N P S I		TNPSI = Total number of ψ
49			
61			
1	A O B (1)	CARD(S) D (73–80)	AOB(1) = First panel aspect ratio $(a/b)_1$
13	A O B (2)		AOB(2) = Second panel aspect ratio $(a/b)_2$
25	.		.
37	.		.
49	.		.
61	.		. (Maximum of 20 (a/b)'s)

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	P H I (1)	CARD(S) E 73 80	PHI(1) = ϕ_1 $\phi = N_y/N_x$
13	P H I (2)		PHI(2) = ϕ_2
25	.		.
37	.		.
49	.		.
61	.		. (Maximum of 20 ϕ)
1	V (1)	CARD(S) F 73 80	V(1) = V_1 $V = \pi^2 \psi t_f (c+t_f)/(2 \beta' c_x b^2)$
13	V (2)		V(2) = V_2
25	.		.
37	.		.
49	.		.
61	.		. (Maximum of 20 V's)
1	P S I (1)	CARD(S) G 73 80	PSI(1) = ψ_1 $\psi = \sqrt{E_x E_y}/(1 - \nu_{xy}\, \nu_{yx})$
13	P S I (2)		PSI(2) = ψ_2
25	.		.
37	.		.
49	.		.
61	.		. (Maximum of 20 ψ's)
1		73 80	
13			
25			
37			
49			
61			

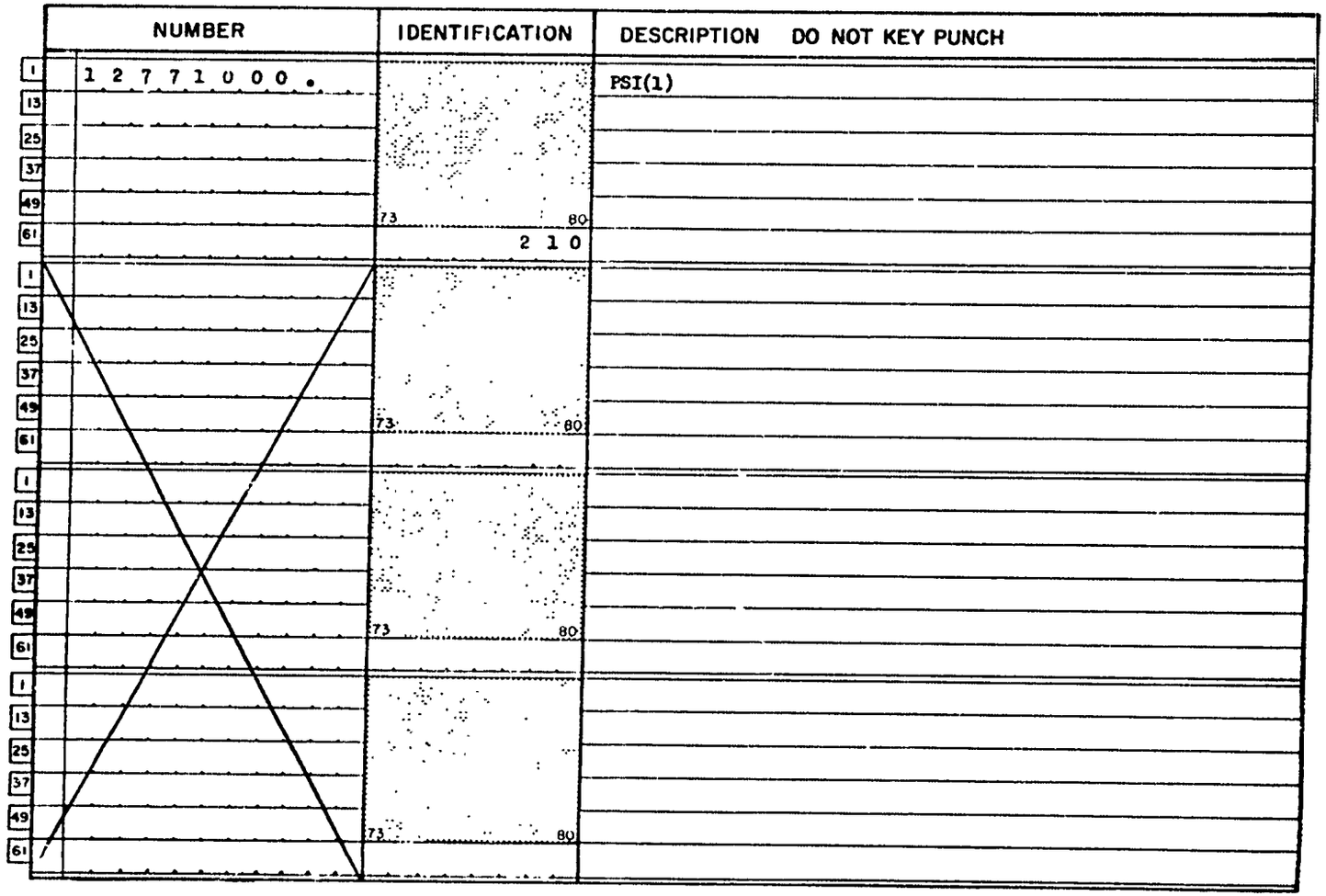
	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	(0/+-45/0)S		TIT [0/±45/0]$_S$ at Room Temperature
13	LAMINATE AT		
25	ROOM TEMPERA		
37	TURE		
49		73 80	
61		10	
1	3.0		TNAOB
13	1.815		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	20	TNV
1	33884000.		AI1
13	128620.		AI2
25	2.0		R
37	1.0		TNPSI
49		73 80	
61		30	
1	1.0		$(a/b)_1$
13	2.0		$(a/b)_2$
25	3.0		$(a/b)_3$
37			
49		73 80	
61		40	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.1		PHI(1)
13	0.5		PHI(2)
25	1.0		PHI(3)
37	2.0		PHI(4)
49	10.0	73 80	PHI(5)
61		50	
1	0.04233		V(1)
13			
25			
37			
49		73 80	
61		60	
1	10794000.		PSI(1)
13			
25			
37			
49		73 80	
61		70	
1	(0/0/+-45/90		TIT $[0_2/\pm45/90_2]_{2S}$ at Room Temperature
13	/90)2S LAMIN		
25	ATE AT ROOM		
37	TEMPERATURE		
49		73 80	
61		80	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3.0		TNAOB
13	1.00		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	90	TNV
1	33884000.		AI1
13	12862 0.		AI2
25	2.0		R
37	1.0		TNPSI
49		73 80	
61		100	
1	1.0		AOB(1)
13	2.0		AOB(2)
25	3.0		AOB(3)
37			
49		73 80	
61		110	
1	0.1		PHI(1)
13	0.5		PHI(2)
25	1.0		PHI(3)
37	5.0		PHI(4)
49	10.0	73 80	PHI(5)
61		120	

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0.18963		V(1)
13			
25			
37			
49		73 80	
61		130	
1	139710000.		PSI(1)
13			
25			
37			
49		73 80	
61		140	
1	(0/+-60)S LA		TIT [0/±60]$_S$ at Room Temperature
13	MINATE AT RO		
25	OM TEMPERATU		
37	RE		
49		73 80	
61		150	
1	3.0		TNAOB
13	1.0		ALF
25	10.0		TNN
37	10.0		TNM
49	5.0	73 80	TNPHI
61	1.0	160	TNV

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
3 3 8 8 4 0 0 0.		AI1
1 2 8 6 2 0.		AI2
2.0		R
1.0		TNPSI
	73 80	
	1 7 0	
1.0		AOB(1)
2.0		AOB(2)
3.0		AOB(3)
	73 80	
	1 8 0	
0.1		PHI(1)
0.5		PHI(2)
1.0		PHI(3)
5.0		PHI(4)
1 0.0	73 80	PHI(5)
	1 9 0	
0.03684		V(1)
	73 80	
	2 0 0	


NUMBER
IDENTIFICATION
DESCRIPTION DO NOT KEY PUNCH
1 2 7 7 1 0 0 0 .
PSI(1)
2 1 0
73
80

```
/ID 00,290,1041,01,DAVIDSON EXT3883                                    12/18/69
/FTC NAME=AC12,LIST,DECK
BPS FORTRAN D COMPILER                                                 12/18/69
                C
                C     AC12...Z- AND HAT-STIFFENED PANEL STABILITY AND STIFFNESS
                C
                C     THE INPUT DATA IS AS FOLLOWS.....
                C           BMAT = ANY DESIRED ONE-CARD TITLE IN COLS. 1-72
                C           A11 = EX*T/(1-MUXY*MUYX)
                C           A12 = MUYX*A11
                C           A16 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
                C           A22 = EY*T/(1-MUXY*MUYX) = A11*EY/EX
                C           A26 = DEFINED BY EQUATION 9 (0.0 FOR SYMMETRIC LAMINATES)
                C           A66 = GXY*T
                C           T = LAMINATE THICKNESS
                C           D11 = SHEET FLEXURAL STIFFNESS IN THE X-DIRECTION FROM
                C                 PROGRAM AC2 OR EQUATION 9
                C           AEXF = FACE SHEET YOUNG'S MODULUS IN THE X-DIRECTION
                C           TP = INDICATOR FOR Z(TP=0.0) AND HAT(TP=1.0)
                C           AK7 = BUCKLING COEFFICIENT FOUND FROM EQUATION 131
                C           AKS = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
                C           AKF = BUCKLING COEFFICIENT FOUND FROM EQUATION 132
                C           AEXS = YOUNG'S MODULUS EX OF THE STIFFENER
                C           TNBS = TOTAL NUMBER OF STIFFENER SPACINGS
                C           TNNX = TOTAL NUMBER OF LOADINGS TO BE CONSIDERED
                C           DSOHS = HAT STIFFENER GEOMETRY RATIO
                C           BNX(I) = LOADINGS
                C           XBS(I) = STIFFENER SPACINGS
                C
S.0001                DIMENSION  BMAT(18),BNX(30),XBS(30)
S.0002              1 READ(5,2) (BMAT(M),M=1,18)
S.0003              2 FORMAT(18A4)
S.0004             45 WRITE(6,46)(BMAT(M),M=1,18)
S.0005             46 FORMAT(1H1,18A4)
S.0006              4 READ(5,3)A11,A12,A16,A22,A26,A66,D11,AEXF,T,TP,AK7,AKS,AKF
S.0007              3 FORMAT(6F12.0)
                C
                C     READ IN  STIFFENER PROPERTIES
                C
S.0008              5 READ(5,3) AEXS,TNBS,TNNX,DSOHS
S.0009              7 IA=TNBS
S.0010                IB=TNNX
S.0011                READ(5,3)(XBS(I),I=1,IA)
S.0012                READ(5,3)(BNX(I),I=1,IB)
```

```
S.0013          DO 200 J=1,IB
S.0014       50 WRITE(6,51)BNX(J),T
S.0015       51 FORMAT(24H0           NX(LB/INCH)=F7.1,7H      T=F7.4)
S.0016       40 WRITE(6,41)
S.0017       41 FORMAT(99H0  BS        HS        TS         FS            01        L,C
                1RIT.       SIGS       SIGF              DS                          )
S.0018          DO 200 I=1,IA
S.0019           ANX=BNX(J)
S.0020            BS=XBS(I)
S.0021          BETA= (AK7/AKS)**.5
S.0022          ALFA =(AEXS*AKF/(AEXF*AKS))**.5
S.0023          IF(TP)17,16,17
                C
                C     LOCAL BUCKLING
                C
                C     CALCULATION OF GEOMETRY OF Z-STIFFENER ELEMENTS TO INSURE
                C     SIMULTANEOUS LOCAL BUCKLING
                C
S.0024       16 HS =(( ANX*AEXS/(AKS*(ALFA*T/BS)**2) -AEXF*T)*BS**2/(ALFA*T *AEXS
                1*(2.*BETA +1.)) )**.5
S.0025          TS=(ALFA * HS * T)/BS
S.0026          FS= BETA *HS
S.0027          GO TO 20
                C
                C     CALCULATION OF GEOMETRY OF HAT-STIFFENED ELEMENTS TO INSURE
                C     SIMULTANEOUS LOCAL BUCKLING
                C
S.0028       17 HS =(( ANX*AEXS/(AKS*(ALFA*T/BS)**2) -AEXF*T)*HS**2/(ALFA*T *AEXS
                1*2.*(1.5+DSOHS)))**.5
S.0029          DS= DSOHS*HS
S.0030          TS= ALFA*HS *T/BS
S.0031          FS=.5* HS
S.0032       20  IF(TP)9,8,9
S.0033        8 AK1=AEXS*TS*(2.*FS+HS)
S.0034          AK2=AEXS*TS*(FS*(HS+.5*T-TS)+.5*HS*(HS+T) +FS*.5*(TS+T))
S.0035          C1=-AEXS*TS*(FS*(HS+.5*T-TS)+ .5*HS*(HS+T) +.5*FS*(TS+T))
S.0036          C2=-AEXS*TS*(FS*(HS*(HS + T -TS) +TS*(TS/3.-T/2.)+T**2/4.) +FS*(T*
                1*2/4. +T*TS/2.+TS**2/3.) +HS*(HS**2/3.+HS*T/2. +T**2/4.))
S.0037          GO TO 10
S.0038        9 AK1= 2.*AEXS*TS*(DS+HS+FS)
S.0039          AK2=AEXS*TS*(DS*(TS+T)+HS*(HS+T) + 2.*FS * (HS +T/2. -TS/2.))
S.0040          C1=- TS*AEXS *( DS*(TS+T) +HS*(HS+T) +2.*FS*(HS +T/2. -TS/2.))
S.0041          C2= -AEXS *2.*TS *( FS*(HS*(HS+T-TS) +TS*(TS/3. -T/4.)+T**2/4. )
                1 +DS*(TS**2/3. +T*TS/2. +T**2/4.)+HS*(HS**2/3. +HS*T/2. +T**2/4.))
```

```
S.0042        10 DEL=(AK2/BS) /( (A12-A16*A26/A66)**2 -(AK1/BS  +A11 -A16**2/A66) *
                1(A22-A26**2/A66))
S.0043           BETA= DEL*(A22-A26**2/A66)
S.0044           D1= -(C1/BS)*BETA-C2/BS + D11
S.0045           ANXL2 = 9.8696*D1
S.0046           XLCR=(ANXL2/ANX)**.5
                 C
                 C     DETERMINATION OF STIFFENER AND SHEET PROPERTIES
                 C
S.0047           IF(TP)14,12,14
S.0048        12 C3= AEXF*T + AEXS*TS*(2.*FS+ HS  -2.*TS)/BS
S.0049           GO  TO  15
S.0050        14 C3= AEXF*T +  AEXS*TS*2.*(DS+HS+  FS)/BS
S.0051        15 SIGF= ANX*AEXF/C3
S.0052           SIGS= ANX*AEXS/C3
S.0053       100 WRITE(6,101)BS,HS,TS,FS,D1,XLCR ,SIGS,SIGF,DS
S.0054       101 FORMAT(F6.3,3F9.3,F14.2,F14.1,2F11.1,F16.3)
S.0055       200 CONTINUE
S.0056           GO TO 1
S.0057           END
                   SIZE OF COMMON  00000    PROGRAM  03644
END OF COMPILATION  AC12
```

ZEE-STIFFENED PANELS WITH (0/+-45/0)S FACES AT ROOM TEMPERATURE

NX(LB/INCH)= 3000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.141	0.011	0.028	798.23	1.6	119171.8	66441.3	0.0
1.250	0.479	0.031	0.094	33676.95	10.5	78528.2	43781.5	0.0
1.500	0.824	0.044	0.162	163845.31	23.2	54954.5	30638.5	0.0
2.000	1.669	0.068	0.328	1221920.00	63.4	30872.5	17212.2	0.0
3.000	4.051	0.109	0.797	15562177.00	226.3	13578.3	7570.3	0.0
4.000	7.378	0.149	1.452	89493424.00	542.6	7579.6	4225.8	0.0

NX(LB/INCH)= 5000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.413	0.033	0.081	28253.52	7.5	124502.4	69413.3	0.0
1.250	0.772	0.050	0.152	170285.88	18.3	80341.7	44792.5	0.0
1.500	1.200	0.065	0.236	593462.00	34.2	55687.5	31047.2	0.0
2.000	2.278	0.092	0.448	3695461.00	85.4	31044.7	17308.2	0.0
3.000	5.347	0.144	1.052	44056144.00	294.9	13600.5	7582.6	0.0
4.000	9.639	0.195	1.897	250316672.00	702.9	7584.8	4228.7	0.0

NX(LB/INCH)= 7500.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.598	0.048	0.118	98522.31	11.4	127350.7	71001.2	0.0
1.250	1.027	0.067	0.202	452445.75	24.4	81280.0	45315.7	0.0
1.500	1.546	0.083	0.304	1447846.00	43.6	56061.3	31255.6	0.0
2.000	2.863	0.116	0.563	8545680.00	106.0	31131.5	17356.6	0.0
3.000	6.619	0.179	1.302	99765376.00	362.3	13611.5	7588.8	0.0
4.000	11.875	0.240	2.337	564602368.00	862.0	7587.4	4230.1	0.0

NX(LB/INCH)= 9999.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.739	0.060	0.145	201218.31	14.1	128823.7	71822.4	0.0
1.250	1.230	0.080	0.242	851455.25	29.0	81757.4	45581.8	0.0
1.500	1.828	0.099	0.360	2647926.00	51.1	56250.1	31360.8	0.0
2.000	3.347	0.135	0.659	15340545.00	123.1	31175.1	17380.9	0.0
3.000	7.682	0.207	1.512	177757312.00	418.9	13617.1	7591.9	0.0
4.000	13.751	0.278	2.706	1004529664.00	995.8	7588.6	4230.9	0.0

HAT-STIFFENED PANELS WITH (/+-45/0)S FACES AT ROOM TEMPERATURE

NX(LB/INCH)= 3000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.062	0.005	0.031	256.98	0.9	117951.9	65761.1	0.131
1.250	0.210	0.014	0.105	5075.68	4.1	75489.1	42087.1	0.445
1.500	0.361	0.020	0.181	25235.45	9.1	52423.1	29227.2	0.767
2.000	0.732	0.030	0.366	209095.13	26.2	29488.0	16440.3	1.553
3.000	1.777	0.048	0.889	3024526.00	99.8	13105.8	7306.8	3.769
4.000	3.236	0.065	1.618	18428448.00	246.2	7372.0	4110.1	6.864

NX(LB/INCH)= 5000.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.181	0.015	0.090	4378.36	2.9	117952.0	65761.2	0.384
1.250	0.339	0.022	0.169	26896.40	7.3	75489.3	42087.2	0.718
1.500	0.526	0.028	0.263	99692.06	14.0	52423.1	29227.2	1.116
2.000	0.999	0.040	0.500	683791.38	36.7	29488.0	16440.3	2.120
3.000	2.345	0.063	1.173	8964109.00	133.0	13105.8	7306.8	4.975
4.000	4.228	0.086	2.114	53027760.00	323.5	7372.0	4110.1	8.968

NX(LB/INCH)= 7500.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.262	0.021	0.131	15536.93	4.5	117951.9	65761.1	0.557
1.250	0.451	0.029	0.225	76094.06	10.0	75489.1	42087.1	0.956
1.500	0.678	0.037	0.339	258790.50	18.5	52423.2	29227.2	1.439
2.000	1.256	0.051	0.628	1656045.00	46.7	29488.0	16440.3	2.664
3.000	2.903	0.078	1.452	20805936.00	165.5	13105.8	7306.8	6.158
4.000	5.209	0.105	2.604	121386912.00	399.7	7372.0	4110.1	11.048

NX(LB/INCH)= 9999.0 T= 0.0416

BS	HS	TS	FS	D1	L,CRIT.	SIGS	SIGF	DS
1.000	0.324	0.026	0.162	32884.21	5.7	117951.9	65761.1	0.687
1.250	0.540	0.035	0.270	149326.00	12.1	75489.2	42087.1	1.145
1.500	0.802	0.043	0.401	490950.56	22.0	52423.1	29227.2	1.701
2.000	1.468	0.059	0.734	3048600.00	54.9	29488.0	16440.3	3.114
3.000	3.370	0.091	1.685	37548640.00	192.5	13105.8	7306.8	7.148
4.000	6.032	0.122	3.016	217602960.00	463.5	7372.0	4110.1	12.794

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13 ABSTRACT

This volume summarizes that portion of the program concerned with the development of theoretical methods of analysis for orthotropic laminated composite structure. An appendix to this volume presents a number of computer programs that were generated in conjunction with these theoretical developments. The main body of this volume is comprised of independent sections devoted to the following topics: extensional and flexural elastic constants, flat plates, orthotropic sandwich plates and shells stiffened skin construction, minimum weight analysis, thermoelastic relationships, and joint analysis.

The appendix presents, for each computer program, an abstract, data input instructions, output description, and a typical example problem. The listed programs cover the following subjects: calculation of laminate extensional and flexural elastic constants; honeycomb sandwich panels under in-plane biaxial loading, in-plane shear loading, normal pressure, and certain combined loadings; hat- and Z-stiffened panel stability and stiffness; and laminate thermoelastic analysis.

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Fiber-Reinforced Materials						
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